SMT1 Solution

Q1 a) -2 b) -1 c) DNE d) 3 e) -1

Q2 a) $\lim\limits_{x\to 3} \dfrac{(x-3)}{(x-3)(x+2)} = \lim\limits_{x\to 3} \dfrac{1}{x+2} = \dfrac{1}{5}$

b) $\lim\limits_{x\to\infty} \dfrac{\sqrt{3(-x)^2-2}}{2(-x)-1} = \lim\limits_{x\to\infty} \dfrac{\sqrt{3x^2-2}\,/x}{(-2x-1)/x} = \lim\limits_{x\to\infty} \dfrac{\sqrt{3-\underbrace{2/x^2}_{\to 0}}}{-2-\underbrace{1/x}_{\to 0}} = -\dfrac{\sqrt{3}}{2}$

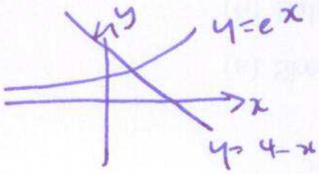
c) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{3x}$ $\quad\theta = 2x$ $\quad = \lim\limits_{\theta\to 0} \dfrac{\sin\theta}{3\theta/2} = \lim\limits_{\theta\to 0} \dfrac{2}{3}\dfrac{\sin\theta}{\theta} = \dfrac{2}{3}.$

d) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{x+4}-2}\cdot\dfrac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim\limits_{x\to 0} \dfrac{x(\sqrt{x+4}+2)}{x+4-4} = \lim\limits_{x\to 0} \sqrt{x+4}+2 = 4$

Q3 $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \dfrac{1}{x-1} = 1$ $\quad \lim\limits_{x\to 2^+} \dfrac{2\cos(\pi x)}{x} = \dfrac{2}{2} = 1 \Rightarrow$ f is cts when $c = 1$

Q4 average rate of change: $\dfrac{f(3)-f(2)}{3-2} = \dfrac{\frac{4}{3}\pi\cdot 3^3 - \frac{4}{3}\pi 2^3}{1} = \frac{4}{3}\pi(27-8) = \frac{4}{3}\pi\cdot 19.$

Q5



$f(x) = e^x - 4 + x$

$\left.\begin{array}{l} f(0) = -3 < 0 \\ f(4) = e^4 > 0 \end{array}\right\}$ IVT $\Rightarrow \exists\ c \in [0,4]$ s.t. $f(c) = 0$

$\therefore\ e^c = 4 - c.$

Q7 a) $f'(2) = \lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} \dfrac{(2+h)^2+(2+h)-4-2}{h} = \lim\limits_{h\to 0} \dfrac{\cancel{4}+4h+h^2+\cancel{2}+h-\cancel{6}}{h}$

$= \lim\limits_{h\to 0} 5+h = 5$

b) $f'(2) = \lim\limits_{h\to 0} \dfrac{\frac{1}{2+h+1}-\frac{1}{2+1}}{h} = \lim\limits_{h\to 0} \dfrac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim\limits_{h\to 0} \dfrac{3-(3+h)}{h.3.(3+h)} = \lim\limits_{h\to 0} \dfrac{-h}{h.3(3+h)}$

$= \lim\limits_{h\to 0} \dfrac{-1}{3(3+h)} = -\dfrac{1}{9}$

c) $f'(2) = \lim\limits_{h\to 0} \dfrac{\sqrt{1+2+h}-\sqrt{1+2}}{h} = \lim\limits_{h\to 0} \dfrac{1}{h}\dfrac{(\sqrt{3+h}-\sqrt{3})(\sqrt{3+h}+\sqrt{3})}{\sqrt{3+h}+\sqrt{3}} = \lim\limits_{h\to 0} \dfrac{3+h-3}{h(\sqrt{3+h}+\sqrt{3})}$

$= \lim\limits_{h\to 0} \dfrac{1}{\sqrt{3+h}+\sqrt{3}} = \dfrac{1}{2\sqrt{3}}$

Q8 a) $20x^4 - 6x^2$

b) $f(x) = x^{5/2} - 2x^{-3/4}$

$f'(x) = \frac{5}{2}x^{3/2} + \frac{6}{4}x^{-7/4}$

c) $8x^3 e^x + 2x^4 e^x$

d) $\dfrac{(2-4x)(5x-2)' - (5x-2)(2-4x)'}{(2-4x)^2}$

$= \dfrac{10-20x+20x-8}{(2-4x)^2} = \dfrac{2}{(2-4x)^2}$

e) $(\sin(x).\sin(x))' = (\sin x)'\sin x + \sin x(\sin x)' = 2\cos x\sin x$

f) $\dfrac{(3-\sin(x)).(4-\sqrt{2}x^{1/2})' - (4-\sqrt{2}x^{1/2})(3-\sin(x))'}{(3-\sin(x))^2} = \dfrac{(3-\sin(x))(-\sqrt{2}.\frac{1}{2}x^{-1/2}) - (4-\sqrt{2}x^{1/2})(-\cos x)}{(3-\sin(x))^2}$

g) $3x^2(e^x\cos x) + x^3(e^x\cos(x))' - 3x^2e^x\cos x + x^3e^x\cos x + x^3e^x.(-\sin x).$

Q9 a) $80x^3 - 12x$

b) $\frac{15}{4}x^{1/2} - \frac{42}{16}x^{-11/4}$

c) $24x^2e^x + 8x^3e^x + 8x^3e^x + 2x^4e^x.$

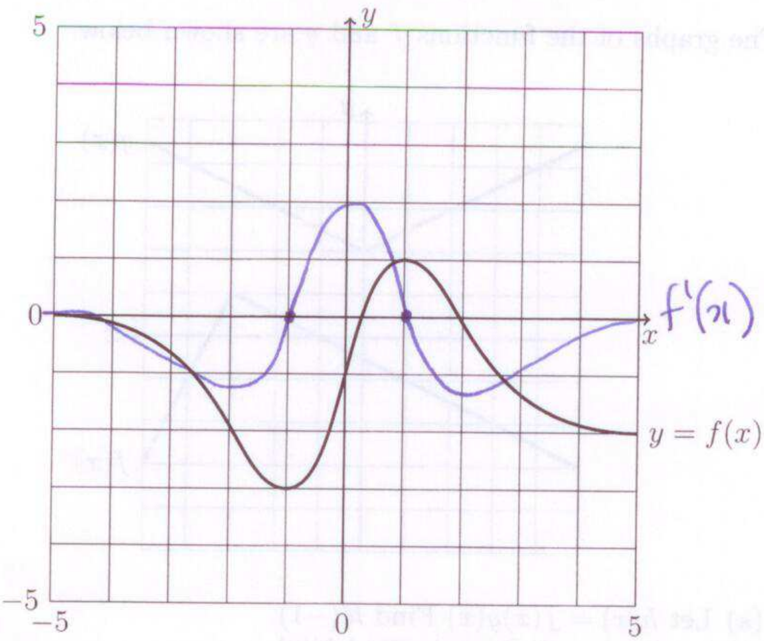
e) $-2\sin x.\sin x + 2\cos x.\cos x.$

Q10a) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h^{-1}(-1) = +\frac{1}{2}.\frac{5}{2} + (-1).(-\frac{1}{2}) = \frac{5}{4} + \frac{1}{2} = \frac{7}{4}.$

b) $h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \Rightarrow h'(2) = \dfrac{3.\frac{1}{2} - \frac{1}{2}.\frac{1}{2}}{(1/2)^2} = \dfrac{\frac{3}{2} - \frac{1}{4}}{\frac{1}{4}} = 6-1 = 5.$

(6) Consider the function $f(x)$ defined by the following graph.



(a) Label all regions where $f'(x) < 0$. $(-\infty,-1) \cup (1,\infty)$

(b) Label all regions where $f'(x) > 0$. $(-1,1)$

(c) Sketch a graph of $f'(x)$ on the figure.

(d) What is $\lim_{x \to \infty} f(x)$? -2

(e) What is $\lim_{x \to -\infty} f'(x)$? 0

(7) Use the limit definition of the derivative to evaluate $f'(2)$, where

(a) $f(x) = x^2 + x$

(b) $f(x) = \dfrac{1}{x+1}$

(c) $f(x) = \sqrt{1+x}$

(8) Find the derivatives of the following functions

(a) $4x^5 - 2x^3$

(b) $\sqrt{x^5} - 2\sqrt[4]{1/x^3}$

(c) $2x^4 e^x$

(d) $\dfrac{5x-2}{2-4x}$

(e) $\sin^2(x)$

(f) $\dfrac{4-\sqrt{2x}}{3-\sin(x)}$

(g) $x^3 e^x \cos(x)$