

**Math 231 Calculus 1 Spring 26 Sample Final**

(1) Differentiate the following functions. Do not simplify your answers.

(a)  $3x^4 - \frac{2}{\sqrt[5]{x^2}} + \sec(x)$

(b)  $f(x) = \frac{x^2 - 2x}{\ln(3 + 2x)}$

(c)  $f(x) = e^{-2x} \cos(3x - 2)$

(d)  $f(x) = \sqrt[3]{e^{\cos(2x)} - 3}$

(2) Evaluate the following integrals.

(a)  $\int \frac{2}{x^3} - 2 \cos(x) - e^x \, dx$

(b)  $\int \frac{(2x - 3)^2}{\sqrt{x^3}} \, dx$

(c)  $\int_0^{\pi/6} \sin(2x) \cos^3(2x) \, dx$

(d)  $\int \frac{1}{1 + 4x^2} \, dx$

(3) Note: the possible answers for limits are a number,  $+\infty$ ,  $-\infty$  or “does not exist” (DNE). Justify your answers.

(a) Find  $\lim_{x \rightarrow 2} \frac{x - 2}{x^2 + x - 6}$ .

(b) Find  $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{\sin(3x)}$ .

(c) Find  $\lim_{x \rightarrow 0^+} x^{\sin(4x)}$ .

(d) Find  $\lim_{x \rightarrow 0} \frac{1}{x^2} - \frac{1}{\sin^2(x)}$ .

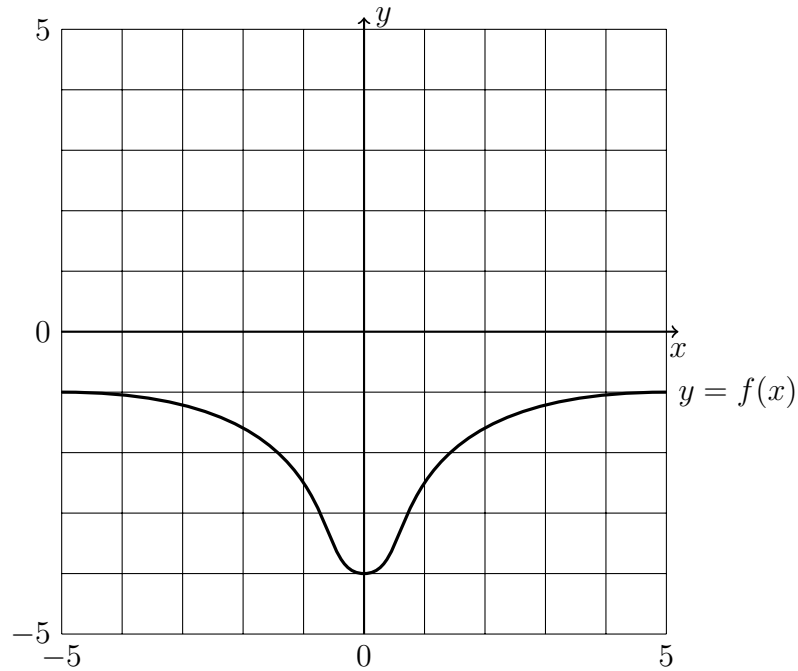
(4) Consider  $f(x) = 12x - x^3$ .

(a) Find the derivative of  $f(x)$ , and find the critical points for  $f(x)$ .

(b) Give the interval(s) for which  $f$  is increasing.

- (c) Give the intervals for which  $f$  is concave up, and for which it is concave down.
- (d) Decide which critical points are maxima, minima, or neither.
- (e) Sketch the graph of  $f(x)$ .

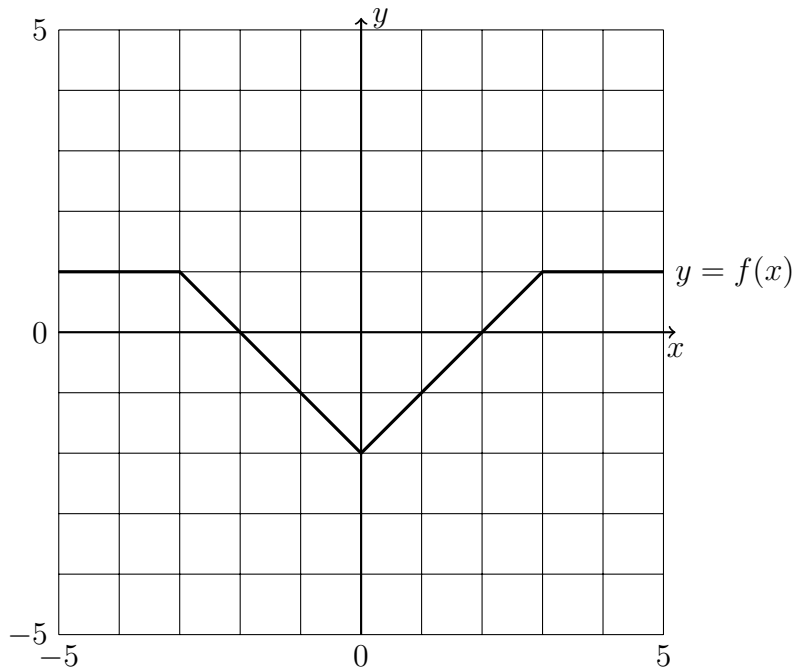
(5) Consider the function  $f(x)$  defined by the following graph.



- (a) Label all regions where  $f(x) < 0$ .
  - (b) Label all regions where  $f'(x) > 0$ .
  - (c) Sketch a graph of  $f'(x)$  on the figure.
- (6) Consider  $f(x) = \frac{1}{x-2}$ .
- (a) Sketch the graph of  $f(x)$  showing any asymptotes.
  - (b) Find the slope of the tangent line at  $x = -1$ , and write down the equation for the tangent line.
  - (c) Sketch the tangent line at  $x = -1$  on your graph.
- (7) Let  $f(x) = \sqrt{x}$ . Find the derivative *using the limit definition of the derivative*. Do not use L'Hôpital's rule. Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation  $x^2y^3 - 2x + 3y = 11$  at the point  $(-2, 1)$ .

(9) Sketch the graph of  $\int_{-5}^x f(t)dt$ , where  $f(x)$  is shown below.



(10) A region in the plane is bounded by the  $x$ -axis, the graph  $y = x^2 + 2$ , and the lines  $x = -2$  and  $x = 1$ .

- Sketch the region (shading it in) and label the boundaries.
- Find the area of the region.

(11) You blow up a spherical balloon at the rate of  $3\text{cm}^3/\text{s}$ . How fast is the radius growing when  $r = 2\text{cm}$ ? (The volume of a sphere is  $V = \frac{4}{3}\pi r^3$ .)

(12) Use linear approximation to estimate  $\sqrt[3]{28}$ . Write down expressions for the absolute and percentage errors.

(13) What's the closest point on the line  $y = 2x + 1$  to the point  $(1, -1)$ ?