SF Solutions

Q1 a) $12x^3 - 2 \cdot \frac{-2}{5} x^{-7/2} + \sec(x)\cot(x)$

b) $$\frac{\ln(3+2x) \cdot (2x-2) - (x^2 - 2x)\frac{1}{3+2x}}{(\ln(3+2x))^2}$$

c) $-2e^{-2x}\cos(3x-2) + e^{-2x} \cdot -\sin(3x-2) \cdot 3.$

d) $\frac{1}{3}\left(e^{\cos(2x)} - 3\right)^{-2/3} \cdot e^{\cos(2x)} \cdot -\sin(2x) \cdot 2$

Q2 a) $\frac{2 \cdot x^{-2}}{-2} - 2\sin(x) - e^x + c$

b) $\int \frac{4x^2 - 12x + 9}{x^{3/2}} dx = \int 4x^{1/2} - 12x^{-1/2} + 9x^{-3/2} dx = 4 \cdot x^{3/2} \cdot \frac{2}{3} - 24x^{1/2} - 9 \cdot x^{-1/2} \cdot 2 + c$

c) $\int_0^{\pi/6} \sin(2x) \cdot \cos^3(2x)\, dx$ $\quad u = \cos(2x)$, $\frac{du}{dx} = -\sin(2x) \cdot 2$ $\quad \int_1^{1/2} \sin(2x) \cdot u^3 \cdot \frac{dx}{du} du = \int_1^{1/2} \sin(2x) \cdot u^3 \cdot \frac{1}{-\sin(2x) \cdot 2} du$

$= \frac{1}{2}\int_{1/2}^{1} u^3 du = \frac{1}{2}\left[\frac{1}{4}u^4\right]_{1/2}^{1} = \frac{1}{8}\left(1 - \frac{1}{16}\right)$

d) $\int \frac{1}{1+4x^2} dx$ $\quad u = 2x$, $\frac{du}{dx} = 2$ $\quad \int \frac{1}{1+u^2} \frac{dx}{du} du = \frac{1}{2}\int \frac{1}{1+u^2} du = \frac{1}{2}\tan u + c = \frac{1}{2}\tan(2x) + c$

Q3 a) $\overset{L'H}{=} \lim_{x \to 2} \frac{1}{2x+1} = \frac{1}{5}$

b) $\overset{L'H}{=} \lim_{x \to 0} \frac{4e^{4x}}{\cos(3x) \cdot 3} = \frac{4}{3}.$

c) $\lim_{x \to 0+} e^{\sin(4x)\ln(x)} = \cancel{\lim}\, e^{\lim_{x \to 0+} \sin(4x)\ln(x)}.$ $\circledast$

$\lim_{x \to 0+} \frac{\ln(x)}{\operatorname{cosec}(4x)} = \lim_{x \to 0} \frac{1/x}{-\operatorname{cosec}(4x) \cdot \cot(4x) \cdot 4} = \lim_{x \to 0} \frac{\sin(4x)\tan(4x)}{4x}$

$= \underbrace{\lim_{x \to 0} \frac{\sin 4x}{4x}}_{=1.} \cdot \underbrace{\lim_{x \to 0} \tan(4x)}_{=0}$ $\quad$ so $\circledast = e^0 = 1.$

d) $$\lim_{x\to0}\frac{1}{x^2}-\frac{1}{\sin^2x}=\lim_{x\to0}\frac{\sin^2x-x^2}{x^2\sin^2(x)}\overset{l'H}{=}\lim_{x\to0}\frac{2\sin x\cos x-2x}{2x\sin^2x+x^2\,2\sin x\cos x}$$

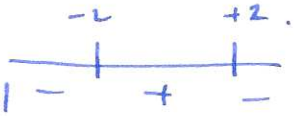
$$=\lim_{x\to0}\frac{\sin2x-2x}{2x\sin^2x+x^2\sin2x}\overset{l'H}{=}\lim_{x\to0}\frac{2\cos2x-2}{2\sin^2x+2x\underbrace{2\sin x\cos x}_{\sin2x}+2x\sin2x+x^2\cos2x\cdot2}.$$

$$=\lim_{x\to0}\frac{\cos2x-1}{\sin^2x+2x\sin2x+x^2\cos2x}\overset{l'H}{=}\lim_{x\to0}\frac{-\sin2x\cdot2}{\underbrace{2\sin x\cos x}_{\sin2x.}+2x\sin2x+2x\cdot\cos2x\cdot2+2x\cos2x+x^2\cdot-\sin2x\cdot2}$$

$$=\lim_{x\to0}\frac{-2\sin2x}{3\sin2x+6x\cos2x-2x^2\sin2x}\overset{l'H}{=}\lim_{x\to0}\frac{-2\cos2x\cdot2}{6\cos2x+6\cos2x+6x\cdot-\sin2x\cdot2-4x\sin2x-2x^2\cdot\cos2x\cdot2}=\frac{-4}{12}=-\frac{1}{3}.$$

Q4 a) $f'(x)=12-3x^2=3(x-2)(x+2)$.

critical points solve $f'(x)=0$: $12-3x^2=0$ $x^2=4$ $x=\pm2$

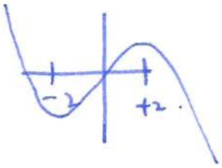


b) increasing $(-2,2)$ decreasing $(-\infty,-2)\cup(2,\infty)$

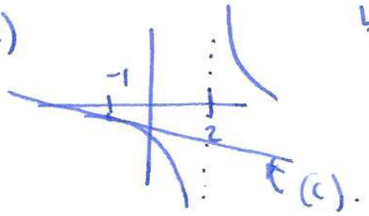
c) $f''(x)=-6x$ concave up $f''(x)>0$ $(-\infty,0)$, concave down $f''(x)<0$ $(0,\infty)$

d) -2 local min $+2$ local max

e)



Q6 a)



b) $f'(x)=-(x-2)^{-2}$ $f'(-1)=-\frac{1}{9}$.

tangent line at $x=-1$ $y+\frac{1}{3}=-\frac{1}{9}(x-1)$

Q7 $$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h\quad(\sqrt{x+h}+\sqrt{x})}.$$

$$=\lim_{h\to0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}.$$

Q8 $x^2y^3-2x+3y=11$

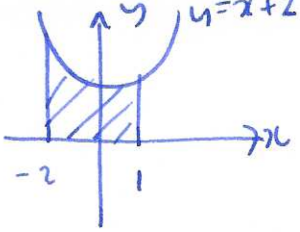
$2xy^3+x^2\,3y^2y'-2+3y'=0$

at $(-2,1)$: $-4+12y'-2+3y'=0$ $y'=\frac{6}{15}=\frac{2}{5}$

$y-1=\frac{2}{5}(x+2)$.

Q10

a)



b) $\int_{-2}^{1} x^2+2\,dx = \left[\frac{1}{3}x^3+2x\right]_{-2}^{1} = \frac{1}{3}+2-\left(-\frac{8}{3}-4\right) = 9$

Q11 $V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\quad \frac{dV}{dt} = 3.$ $\quad \frac{dr}{dt} = \frac{3}{4\pi . 4} = \frac{3}{16\pi}$ cm/s.

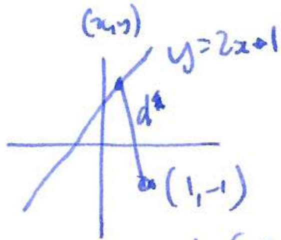
Q12 $f(x) = \sqrt[3]{x} = x^{1/3}.$ $\quad f'(x) = \frac{1}{3}x^{-2/3}$

$f(27) = 3$

$f'(27) = \frac{1}{3.9} = \frac{1}{27}.$

$f(28) \approx f(27) + 1.f'(27) = 3 + \frac{1}{27}.$

Q13



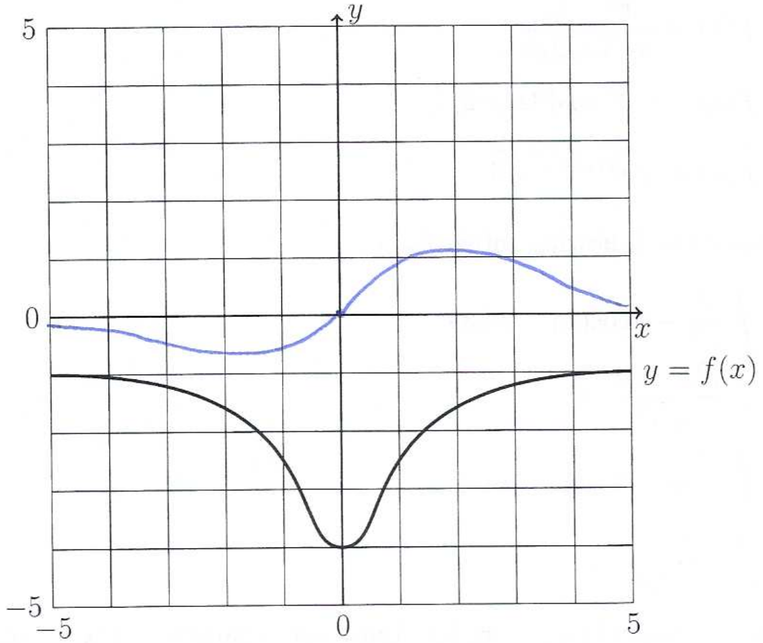
$d^2 = (x-1)^2 + (y+1)^2 = (x-1)^2 + (2x+1+1)^2 = (x-1)^2 + (2x+2)^2$

$\frac{d}{dx}(d^2) = 2(x-1) + 2(2x+2).2 = 10x+6$

critical point $\frac{d}{dx}(d^2) = 0$: $\quad x = -\frac{6}{10} = -\frac{3}{5}$ $\quad y = -\frac{1}{5}.$

(c) Give the intervals for which f is concave up, and for which it is concave down.
(d) Decide which critical points are maxima, minima, or neither.
(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.



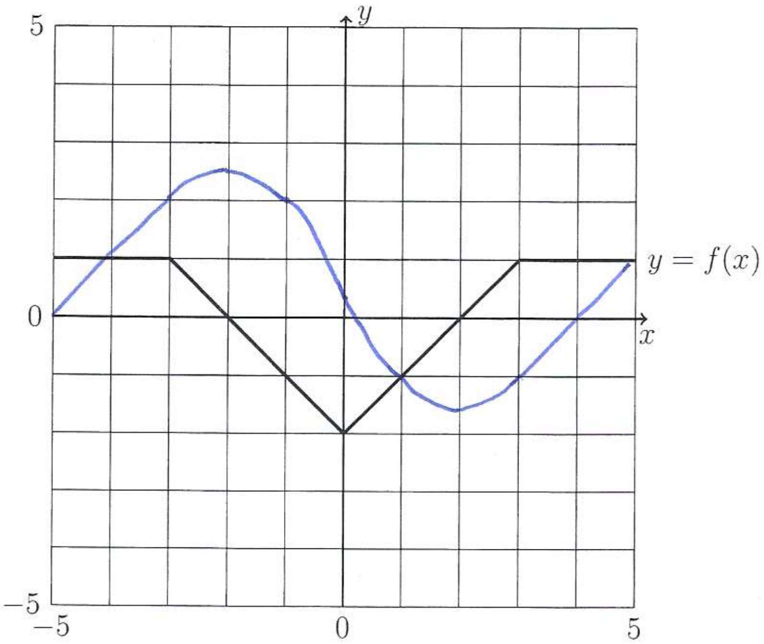
(a) Label all regions where $f(x) < 0$. [illegible] $[-5,5]$
(b) Label all regions where $f'(x) > 0$. $(0,5)$
(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{1}{x-2}$.
(a) Sketch the graph of $f(x)$ showing any asymptotes.
(b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.
(c) Sketch the tangent line at $x = -1$ on your graph.

(7) Let $f(x) = \sqrt{x}$. Find the derivative *using the limit definition of the derivative.* Do not use L'Hôpital's rule. Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y^3 - 2x + 3y = 11$ at the point $(-2, 1)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.



(10) A region in the plane is bounded by the x-axis, the graph $y = x^2 + 2$, and the lines $x = -2$ and $x = 1$.
- (a) Sketch the region (shading it in) and label the boundaries.
- (b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $3\text{cm}^3/\text{s}$. How fast is the volume growing when $r = 2\text{cm}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

(12) Use linear approximation to estimate $\sqrt[3]{28}$. Write down expressions for the absolute and percentage errors.

(13) What's the closest point on the line $y = 2x + 1$ to the point $(1, -1)$?