useful facts if the Taylor series converges, we can:

- differentiate it
- integrate it
- multiply (xe^x)
- substitute (e^{x^2})

Examples xe^x, e^{-x^2}, $e^x \sin x$

Fact works for complex numbers $x+iy$, $e^{i\theta} = \cos\theta + i\sin\theta$.

get radius of converge R in complex plane, explains why $\frac{1}{1+x^2}$ power series has radius of convergence 1, as pole/singularity at $x = \pm i$.

How to make l'Hôpital's rule problems:

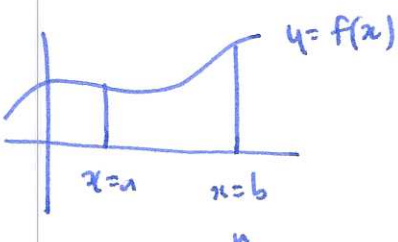
$$\left.\begin{aligned} \sin(2x) &\approx 2x - \frac{(2x)^3}{3!} + o(x^5) \\ e^{3x} &\approx 1 + 3x + o(x^2) \end{aligned}\right\} \quad \frac{\sin(2x)}{e^{3x}} \approx \frac{2x}{3x} = \frac{2}{3}.$$

Cheap error bounds for e^x: $(x<1)$

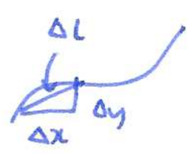
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \underbrace{\frac{x^{n+1}}{(n+1)!} + \cdots}_{\leq x^{n+1} + x^{n+2} + \cdots = \frac{x^{n+1}}{1-x}}$$

so $e^{1/2} = 1 + \frac{1}{2} + \frac{(1/2)^2}{2!} + \cdots + \frac{(1/2)^n}{n!}$ with error $\leq \frac{1}{2^{n+1}}$

§8.1 Arc length and surface area



Q: how long? A.



$$|\Delta L| \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\text{length} = \sum_{i=1}^{n} |\Delta L_i| = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sum_{i=1}^{n} \sqrt{1 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}}\, \Delta x_i$$

take limit as $|\Delta x_i| \to 0$: arc length $= \int_a^b \sqrt{1 + (f'(x))^2}\, dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$