<u>even powers</u> $\int \sin^4 x \cos^2 x \, dx$ ← get everything in terms of $\sin x$ or $\cos x$, then use parts.

$$\int \sin^4 x (1-\sin^2 x) \, dx = \int \sin^4 x - \sin^6 x \, dx$$

$$\int \sin^6 x \, dx = \int \underset{u}{\sin^5 x} \, \underset{v'}{\sin x} \, dx \qquad u = \sin^5 x \quad v' = \sin x$$
$$u' = 5\sin^4 x \cos x \quad v = -\cos x$$

$$= uv - \int u'v \, dx$$

$$= -\sin^5 x \cos x + \int 5 \sin^4 x \cos^2 x \, dx$$

$$= -\sin^5 x \cos x + 5 \int \sin^4 x (1-\sin^2 x) \, dx$$

$$\int \sin^6 x \, dx = -\sin^5 x \cos x + 5 \int \sin^4 x \, dx - 5 \int \sin^6 x \, dx$$

$$6 \int \sin^6 x \, dx = -\sin^5 x \cos x + \underbrace{5 \int \sin^4 x \, dx}$$

do by parts!

<u>other trig functions</u>

<u>recall</u>: $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx \qquad u = \cos x, \; \frac{du}{dx} = -\sin x \qquad = \int \frac{\sin x}{u} \cdot \frac{-1}{\sin x} du$

$$= -\int \frac{1}{u} du = -\ln|u| + c = -\ln|\cos x| + c = +\ln|\sec x| + c$$

<u>fact</u>: $\int \sec x \, dx = \ln|\sec x + \tan x| + c \qquad$ check: $\frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$

$\int \operatorname{cosec} x \, dx = -\ln|\operatorname{cosec} x + \cot x| + c$

<u>other trig function powers</u>

$\int \tan^a x \sec^b x \, dx \qquad$ use:

- $\cos^2 x + \sin^2 x = 1 \iff 1 + \tan^2 x = \sec^2 x$
- $u = \sec x \quad \frac{du}{dx} = \sec x \tan x$
- $u = \tan x \quad \frac{du}{dx} = \sec^2 x$
- parts

a odd : $\int \tan^3 x \sec^2 x \, dx = \int \tan^2 x \sec x \, (\tan x \sec x) \, dx$

$= \int (1 - \sec^2 x) \sec x \, (\tan x \sec x) \, dx$ $\quad u = \sec x$, $\frac{du}{dx} = \sec x \tan x$

$= \int (1-u^2) \, u \, du = \frac{1}{2} u^2 - \frac{1}{4} u^4 + c = \frac{1}{2} \sec^2 x - \frac{1}{4} \sec^4 x + c$

b even : $\int \tan^3 x \sec^2 x \, dx$ $\quad u = \tan x$, $\frac{du}{dx} = \sec^2 x$

$\int u^3 \cdot \frac{\sec^2 x}{\sec^2 x} \, du = \frac{1}{4} u^4 + c = \frac{1}{4} \tan^4 x + c$

a even, b odd : write as powers of sec(x) and use integration by parts.

Example $\int \sin(3x) \cos(2x) \, dx$ $\quad$ useful fact: $\sin(A+B) = \sin A \cos B + \cos A \sin B$ ①
$\sin(A-B) = \sin A \cos B - \cos A \sin B$ ②

① + ② : $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

$\int \sin 3x \cos 2x \, dx = \frac{1}{2} \int \sin 5x + \sin x \, dx = -\frac{1}{10} \cos 5x - \frac{1}{2} \cos x + c$

Example $\int \cos(4x) \cos(7x) \, dx$ $\quad$ useful fact : $\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

so $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

$= \frac{1}{2} \int \cos 11x + \cos(-3x) \, dx = \frac{1}{22} \sin 11x + \frac{1}{6} \sin 3x + c$

## §7.3 Trig substitutions

aim : deal with $\sqrt{a^2 - x^2}$, $\sqrt{a^2 + x^2}$, $\sqrt{x^2 - a^2}$
① ② ③

$\cos^2 x + \sin^2 x = 1 \leftrightarrow \sin^2 x = 1 - \cos^2 x$ ①

$\leftrightarrow 1 + \tan^2 x = \sec^2 x$ ②

$\leftrightarrow \cot^2 x + 1 = \operatorname{cosec}^2 x \leftrightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$ ③ .