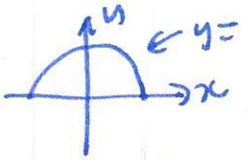
check: surface area of sphere

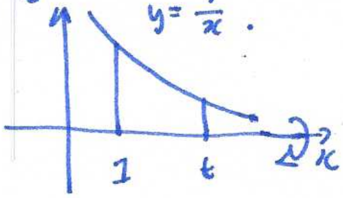


$$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)$$

surface area $S = 2\pi \int_{-1}^{1} \sqrt{1-x^2}\sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)}\,dx$

$= 4\pi.$

<u>Example</u>



between 1 and t, let $t \nearrow \infty$, gives between 1 and ∞.

volume $V = \int_1^t \frac{\pi}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^t = \pi\left(1-\frac{1}{t}\right)$

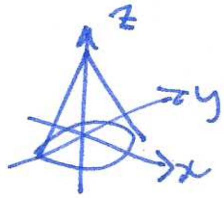
$\to \pi$ as $t \to \infty$.

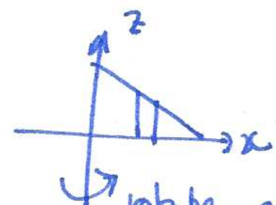
surface area: $A = 2\pi\int_0^t \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx > 2\pi\int_1^t \frac{1}{x}\,dx = 2\pi\left[\ln|x|\right]_1^t$

$= 2\pi \ln(t) \to \infty$ as $t \to \infty$.

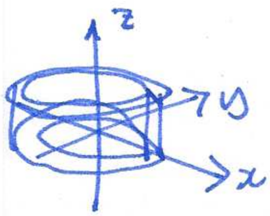
<u>§6.4 Cylindrical shells</u>

volume of cone:





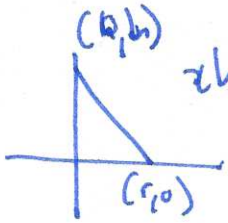
rotate around z-axis.



volume of shell $V_i \approx 2\pi x_i y_i \Delta x_i$

$$V = 2\pi \int_a^b x f(x)\,dx$$

<u>Example</u>



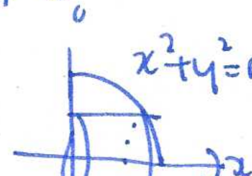
$xh + yr = hr$

$$V = \int_0^r 2\pi x\left(\frac{hr - xh}{r}\right)dx$$

$$V = \frac{2\pi h}{r}\int_0^r x(r-x)\,dx$$

$= 2\pi\frac{h}{r}\int_0^r xr - x^2\,dx = 2\pi\frac{h}{r}\left[\frac{1}{2}x^2 r - \frac{1}{3}x^3\right]_0^r = 2\pi\frac{h}{r}\left(\frac{1}{2}r^3 - \frac{1}{3}r^3\right) = \frac{1}{3}\pi r^2.$

<u>Example</u>



volume of hemisphere: rotate horizontal rectangle about x-axis.

$$V = \int 2\pi y\, f(y)\,dy$$

$V = \int_0^r 2\pi y\sqrt{r^2-y^2}\,dy = 2\pi\left[\frac{2}{3}(r^2-y^2)^{3/2}\cdot -\frac{1}{2}\right]_0^r = \frac{-2\pi}{3}(0-r^2) = \frac{2}{3}\pi r^3.$

§7.1 Integration by parts

"reverse product rule" product rule: $(uv)' = u'v + uv'$

$$\int (uv)'\,dx = \int u'v + uv'\,dx$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\int uv'\,dx = uv - \int u'v\,dx}$$ ← integration by parts formula

Example ① $\int x\sin x\,dx$ $u = x$, $v' = \sin x$; $u' = 1$, $V = -\cos x$

$$\int \underbrace{x}_{u}\underbrace{\sin x}_{v'}\,dx = \underbrace{x}_{u}\underbrace{(-\cos x)}_{v} - \int \underbrace{1}_{u'}\cdot\underbrace{(-\cos x)}_{v}\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + c$$ check!

② $\int xe^x\,dx$ $u = x$, $v' = e^x$; $u' = 1$, $v = e^x$ } Q: spose we chose these the other way round?

$$= uv - \int u'v\,dx = xe^x - \int 1e^x\,dx = xe^x - e^x + c$$

observation: $\int_a^b uv'\,dx = [uv]_b^a - \int_a^b u'v\,dx$

③ $\int_1^2 \ln(x)\,dx = \int_1^2 \underbrace{1}_{v'}\cdot\underbrace{\ln x}_{u}\,dx$ $u = \ln(x)$, $v' = 1$; $u' = \frac{1}{x}$, $v = x$

$$= [x\ln(x)]_1^2 - \int_1^2 x\cdot\frac{1}{x}\,dx$$

$$= 2\ln(2) - \ln(1) - [x]_1^2 = 2\ln(2) - 1$$

(4) $\int x^2\cos(x)\,dx = x^2\sin(x) - \int 2x(-\cos x)\,dx$

$= x^2\sin x + 2x\cos x - 2\sin x + c$

(5) $\int e^x \sin x\,dx = e^x(-\cos x) - \int \underbrace{e^x}_{u}\underbrace{(-\cos x)}_{v'})\,dx$

$\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$

$2\int e^x\sin x\,dx = e^x(\sin x - \cos x)$

$\int e^x\sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + c$

§7.2 Trig integrals $\int \sin^m x\cos^n x\,dx$

tools:
- $\cos^2 x + \sin^2 x = 1$
- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- sub $u = \sin x$ $\frac{du}{dx} = \cos x$
- sub $u = \cos x$ $\frac{du}{dx} = -\sin x$
- parts

Examples
- $\int \sin^2 x\,dx = \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$
- $\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx$ sub $u = \cos x$, $\frac{du}{dx} = -\sin x$

$= \int (1-u^2)\sin x \cdot \frac{1}{-\sin x}\,du = -\int 1 - u^2\,du = -u + \frac{1}{3}u^3 + c$

$= -\cos x + \frac{1}{3}\cos^3 x + c$

moral: squares: double angle formula
odd powers: do sub u = other trig function