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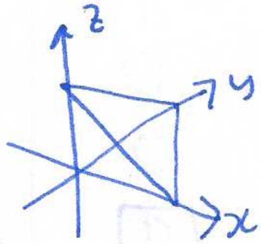
Example tetrahedron with vertices $(0,0,0), (0,0,1), (0,1,0), (1,0,0)$

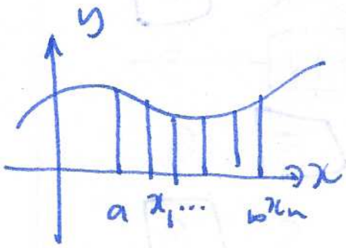


$$V = \int_0^1 A(z)\,dz \qquad \text{area of triangle} = \tfrac{1}{2}\,\text{base} \times \text{height}$$
$$= \tfrac{1}{2}(1-z)(1-z)$$

$$V = \int_0^1 \frac{1}{2}(1-z)^2\,dz = \cdots = \frac{1}{6}$$

recall: average value of numbers $a_1, \ldots, a_n = \dfrac{a_1 + \cdots + a_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} a_i$

Q: what about average value of function $f(x)$ on an interval $[a,b]$?



$$\text{average} \approx \frac{1}{n}\left(f(x_1) + f(x_2) + \cdots + f(x_n)\right)$$

recall: $\displaystyle\int_a^b f(x)\,dx \approx \left(f(x_1) + f(x_2) + \cdots + f(x_n)\right)\Delta x$

$\Delta x = \dfrac{b-a}{n}$

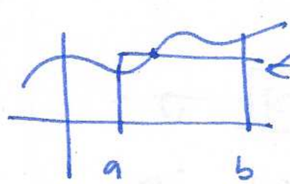
so $\displaystyle\int_a^b f(x)\,dx \approx (\text{average of } f(x))\,(b-a)$ so $\text{average of } f(x) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

Example find average value of $\sin(x)$ on $[0,\pi]$:

$$\frac{1}{\pi}\int_0^{\pi} \sin(x)\,dx = \frac{1}{\pi}\Big[-\cos(x)\Big]_0^{\pi} = \frac{1}{\pi}\left(-(-1)+1\right) = \frac{2}{\pi}$$

useful fact (mean value theorem for integrals)

If $f(x)$ is cts on $[a,b]$, then there is a $c \in [a,b]$ s.t. $f(c) = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

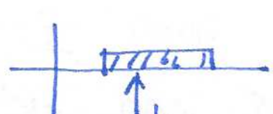


← average value

Example find average value of $\dfrac{\sin(\pi/x)}{x^2}$ on $[1,2]$

Q: which has bigger average on $[0,\pi]$, $\sin(x)$ or $\sin^2 x$?

Application: centers of mass



density $\rho(x)$