MTH 232 Calculus 2

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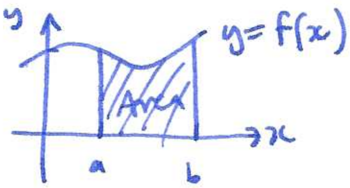
– math tutoring 1S-214
– students w/ disabilities

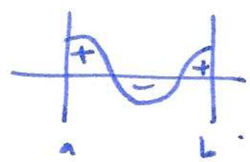
Text: Calculus, early transcendentals, Rogawski + Adams
HW: webworks / projects / quizzes

§5.2 Definite integral



intuition: $\int_a^b f(x)\,dx =$ area under the curve $y = f(x)$ between $x = a$ and $x = b$

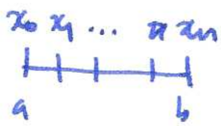
note: <u>signed</u> area



formal def$^{\underline{n}}$: Riemann sum $R(f, P, c) = \sum f(c_i)\Delta x_i$, $\Delta x_i = |x_i - x_{i-1}|$

f function $f: \mathbb{R} \to \mathbb{R}$

P partition of $[a, b]$



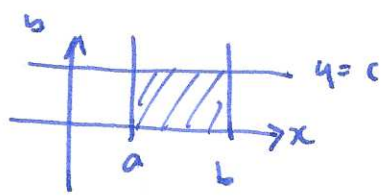
c choice of points $c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\|P\| \to 0} R(f, P, c)$$

$$\|P\| = \max \Delta x_i$$

<u>useful properties</u>

$$\int_a^b c\,dx = c(b - a)$$



$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c f(x)\,dx = c \int_a^b f(x)\,dx$$

reversing limits: $\int_a^b f(x)\,dx = - \int_b^a f(x)\,dx$

0-length intervals : $\int_a^a f(x)\,dx = 0$

adjacent intervals : $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Indefinite integrals

Def$^{\underline{n}}$ A function $F(x)$ is an <u>anti-derivative</u> for $f(x)$ if $F'(x) = f(x)$

<u>General antiderivative</u> if $F(x)$ is an antiderivative for $f(x)$, then any other antiderivative is of the form $F(x)+c$ for some constant c.

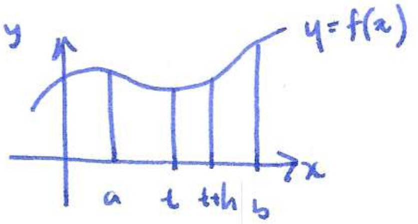
⌈ <u>Proof</u>: suppose f has antiderivatives F, G, then $(F-G)' = F' - G' = f - f = 0$
$\Rightarrow$ $F-G$ is constant function. ⌋

<u>notation</u> : $\int f(x)\,dx = F(x)+c$ means $F(x)+c$ is general antiderivative for $f(x)$.

§5.4 Fundamental theorem of calculus I

Th$^{\underline{m}}$ (FTC①) suppose $f(x)$ is cts on $[a,b]$ and $F(x)$ is an antiderivative for $f(x)$, i.e. $F'(x) = f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

<u>intuition</u> :



consider $\int_a^t f(x)\,dx$ ← function of t !

Q: what is the rate of change wrt t?

<u>recall</u> $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ so $\frac{d}{dt}\left(\int_a^t f(x)\,dx\right)$

$$= \lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{t\to 0} \frac{1}{h} \int_t^{t+h} f(x)\,dx$$

$\approx$ area of rectangle $\frac{f(t)\times h}{h} = f(t)$

i.e. $\int_a^t f(x)\,dx$ is an antiderivative for $f(x)$, so $\int_a^x f(t)\,dt = F(x)+c$

Q: what is the constant? $t=a$ $\int_a^a f(x)\,dx = 0 = F(a)+c \Rightarrow c=-F(a)$

so $\int_a^t f(x)\,dx = F(t)-F(a)$. $\square$

<u>Example</u> $\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{2x^{3/2}}{3} + \ln|x| - \cos(x)\right]_2^3$

$= \frac{2.3^{3/2}}{3} + \ln(3) - \cos(3) - \left(\frac{2.2^{3/2}}{3} + \ln(2) - \cos(2)\right)$

<u>§5.5 Fundamental theorem of calculus II</u>

<u>Thm</u> (FTC(2)) let $f(x)$ be a cts function on $[a,b]$, then $A(x)=\int_a^x f(t)\,dt$ is an antiderivative for $f(x)$, i.e. $A'(x)=f(x)=\frac{dA}{dx} \Leftrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

furthermore $A(a)=0$

<u>Example</u>: $\int_0^x e^{-t^2}\,dx \leftarrow$ a function with derivative e^{-x^2}

<u>Example</u>: what about $\int_0^{x^2} \sin(t)\,dt \leftarrow$ function of a function!

to find $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$ set $A(x)=\int_0^x \sin(t)\,dt$, then $A'(x)=\sin(t)$

so $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt = \frac{d}{dt}\left(A(x^2)\right) \overset{\text{chain rule}}{=} A'(x^2)\,(x^2)' = A'(x^2)\,2x = \sin(x^2).2x$

[aside: when people say "not every function can be integrated" what do they mean? If $f(x)$ is cts then $\int_0^x f(t)\,dt$ is an integral for $f(x)$, but we might not be able to write it out as a formula involving basic functions.

Analogy: • $\sqrt{2} \approx 1.414$ is a real number but not a fraction

• $x^5 - x - 1$ has 1 real root which cannot be written as an expression involving rational numbers and fractional powers [Galois theory]

• e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory]