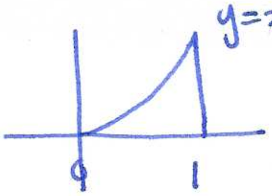
Example 1



$\frac{dy}{dx} = 2x$ arc length $\int_0^1 \sqrt{1+4x^2}\,dx$

(trig sub integral).

2 $f(x) = \frac{1}{12}x^3 + \frac{1}{x}$ on $[1,2]$

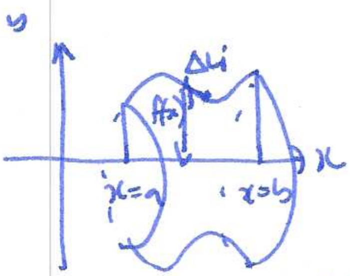
$f'(x) = \frac{1}{4}x^2 - \frac{1}{x^2}$

arc length $= \int_1^2 \sqrt{1+\left(\frac{1}{4}x^2 - \frac{1}{x^2}\right)}\,dx = \int_1^2 \sqrt{1+\frac{1}{16}x^4 - \frac{1}{2} + \frac{1}{x^4}}\,dx$

$= \int_1^2 \sqrt{\frac{1}{16}x^4 + \frac{1}{2} + \frac{1}{x^4}}\,dx = \int_1^2 \sqrt{\left(\frac{1}{4}x^2 + \frac{1}{x^2}\right)^2}\,dx = \int_1^2 \frac{x^2}{4} + \frac{1}{x^2}\,dx$

$= \left[\frac{x^3}{12} - \frac{1}{x}\right]_1^2 = \frac{8}{12} - \frac{1}{4} - \frac{1}{12} + 1 = \frac{4}{3}.$

Area of surface of revolution

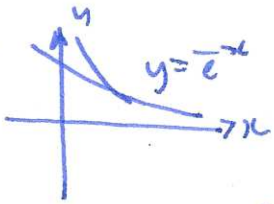


area $\approx \sum_{i=1}^{n} |\Delta L_i|\, f(x)\, 2\pi$ $\qquad |\Delta L_i| = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$.

take limit as $|\Delta x_i| \to 0$

surface area of volume of revolution $= \int_a^b 2\pi f(x) \sqrt{1+f'(x)^2}\,dx$

Example



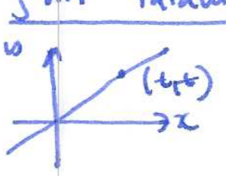
$\frac{dy}{dx} = -e^{-x}$ surface area $= \int_0^\infty 2\pi e^{-x} \sqrt{1+(-e^{-x})^2}\,dx$

sub $u = e^{-x}$

$\frac{du}{dx} = -e^{-x}$

$\int_1^0 2\pi u \sqrt{1+u^2}\,\frac{dx}{du}\,du = \int_1^0 2\pi\sqrt{1+u^2}\,du$ (trig sub integral)

§11.1 Parametric equations

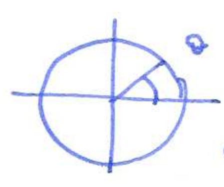


$x(t) = t$

$y(t) = t$

$c(t) = (x(t), y(t))$ parameterization

e.g.



$c(\theta) = (\cos\theta, \sin\theta)$

$c(t) = (\cos t, \sin t)$