Math 232 Calculus 2 Spring 25 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int e^x \sin(1+e^x)\, dx$.

$u = 1+e^x$

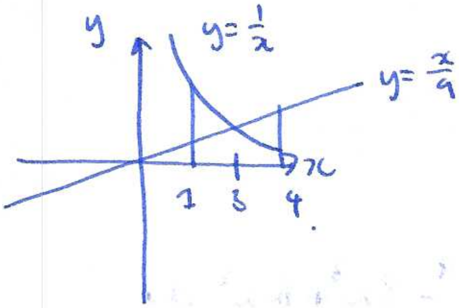
$\frac{du}{dx} = e^x$

$\int e^x \sin(u) \frac{dx}{du} du$

$= \int e^x \sin(u) \frac{1}{e^x} du = \int \sin(u)\, du = -\cos(u) + C = -\cos(1+e^x) + C$

(2) (10 points) Find the area bounded between the curves $y = 1/x$ and $y = x/9$ over the interval $1 \le x \le 4$.

$\frac{1}{x} = \frac{x}{9} \Rightarrow \quad x^2 = 9 \quad x = \pm 3$



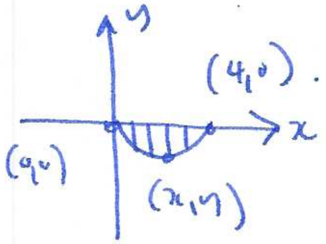
$$\int_1^3 \frac{1}{x} - \frac{x}{9}\,dx + \int_3^4 \frac{x}{9} - \frac{1}{x}\,dx$$

$$= \left[\ln|x| - \frac{x^2}{18}\right]_1^3 + \left[\frac{x^2}{18} - \ln|x|\right]_3^4$$

$$= \ln(3) - \frac{1}{2} - \ln(1) + \frac{1}{18} + \frac{16}{18} - \ln(4) - \frac{1}{2} + \ln(3).$$

$$= 2\ln(3) - \ln(4) - \frac{1}{18}$$

(3) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 4x$ and the x-axis. Find the volume of revolution of this region rotated about the x-axis.

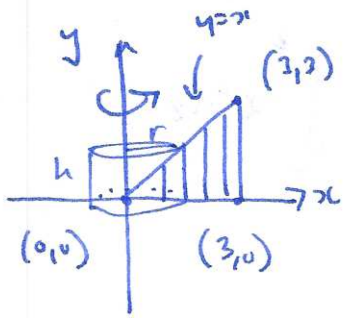


$$V = \int_0^4 \pi r^2 \, dx = \pi \int_0^4 y^2 \, dx$$

$$= \pi \int_0^4 (x^2 - 4x) \, dx = \pi \int_0^4 x^4 - 8x^3 + 16x^2 \, dx$$

$$= \pi \left[\frac{1}{5} x^5 - 2x^4 + \frac{16}{3} x^3 \right]_0^4 = \pi \left(\frac{1024}{5} - 512 + \frac{1024}{3} \right) = \frac{512\pi}{15}$$

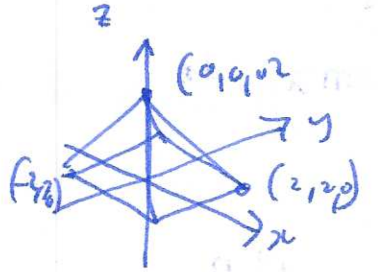
(4) (10 points) Find the volume of revolution of the triangle with vertices $(0, 0)$, $(3, 3)$ and $(3, 0)$, rotated about the y-axis.

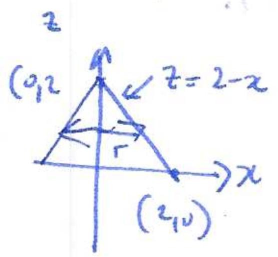


$$V = \int_0^3 A(x)\,dx = \int_0^3 2\pi r h\,dx = \int_0^3 2\pi x y\,dx$$

$$= 2\pi \int_0^3 x^2\,dx = 2\pi \left[\tfrac{1}{3}x^3\right]_0^3 = 18\pi$$

(5) (10 points) Find the volume of the pyramid whose base is the square in the xy-plane with vertices $(\pm 2, \pm 2, 0)$ and whose top point is $(0, 0, 2)$.





$r = 2x$

$$V = \int_0^2 A(z)\,dz = \int_0^2 (r^2)\,dz = \int_0^2 4x^2\,dz$$

$$= 4\int_0^2 (2-z)^2\,dz = 4\int_0^2 4 - 4z + z^2\,dz$$

$$= 4\left[4z - 2z^2 + \frac{1}{3}z^3\right]_0^2 = 4\left(8 - 8 + \frac{1}{3}\cdot 8\right) = \cancel{\frac{8}{3}}\ \frac{32}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(6) (10 points) Find $\int \underbrace{x}_{u}\underbrace{e^{-3x}}_{v'}\,dx.$ $= x \cdot -\frac{1}{3}e^{-3x} - \int 1 \cdot -\frac{1}{3}e^{-3x}\,dx$

$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + c$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(7) (10 points) Find $\int_0^\pi \underbrace{e^{2x}}_{u} \underbrace{\cos(x)}_{v'}\, dx$.

$$\int_0^\pi e^{2x}\cos(x)\,dx = \underbrace{\left[e^{2x} \cdot \sin(x) \right]_0^\pi}_{0} - \int_0^\pi \underbrace{2e^{2x}}_{u} \cdot \underbrace{\sin(x)}_{v'}\,dx$$

$$\int_0^\pi e^{2x}\cos(x)\,dx = -\left[2e^{2x} \cdot -\cos x \right]_0^\pi + \int_0^\pi 4e^{2x} \cdot -\cos(x)\,dx$$

$$5\int_0^\pi e^{2x}\cos(x)\,dx = -2e^{2\pi} - 2$$

$$\int_0^\pi e^{2x}\cos(x)\,dx \quad -\frac{2}{5}(e^{2\pi}+1)$$

(8) Find $\int \cos^3 x \, dx.$ $= \int \cos(x) \frac{\cancel{\sin^2} x}{\cos} \, dx = \int \cos(x) \left(1 - \cancel{\cos}^{\sin^2} x\right) dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$\int \cos(x)(1-u^2) \frac{dx}{du} \, du = \int \cos(x)(1-u^2) \frac{1}{\cos(x)} \, dx$

$= \int 1-u^2 \, du = u - \frac{1}{3}u^3 + c = \sin x - \frac{1}{3}\sin^2 x + c$

(9) Find $\int \cos(4x)\cos(3x)\,dx$.

$\sin(A+B) = \sin A \cos B + \sin \cos A \sin B$
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A \cos B$.

$\frac{1}{2}\int \cos(7x) + \cos(x)\,dx = \frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x) + C$.

(10) Find $\int \frac{3-x}{(x+1)(x-1)^2}\,dx$.

$$\frac{3-x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x+1)(x-1)^2}.$$

$x=-1$: $\quad 4 = 4A \quad A = 1$

$x=1$: $\quad 2 = 2C \quad C = 1$

$x=0$: $\quad 3 = +A - B + C \quad B = -1$

$$\int \frac{1}{x+1} - \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx = \ln|x+1| - \ln|x-1| - \frac{1}{(x-1)} + c.$$