**Math 232 Calculus 2 Spring 25 Midterm 1a**

**Name:** Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a $3 \times 5$ index card of notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 1 | |
| Overall | |

(1) (10 points) Find $\int e^x \cos(1+e^x)\, dx$.

sub $u = 1+e^x$

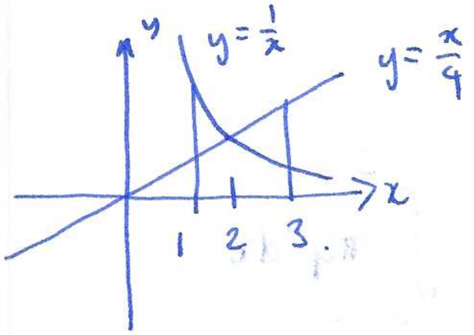
$\frac{du}{dx} = e^x$

$\int e^x \cos(u) \frac{dx}{du}\, du$

$= \int e^x \cos(u) \frac{1}{e^x}\, du = \int \cos(u)\, du = \sin(u) + c$

$= \sin(1+e^x) + c$

(2) (10 points) Find the area bounded between the curves $y = 1/x$ and $y = x/4$ over the interval $1 \leq x \leq 3$.




$\frac{1}{x} = \frac{x}{4} \Rightarrow x^2 = 4$

$x = \pm 2.$

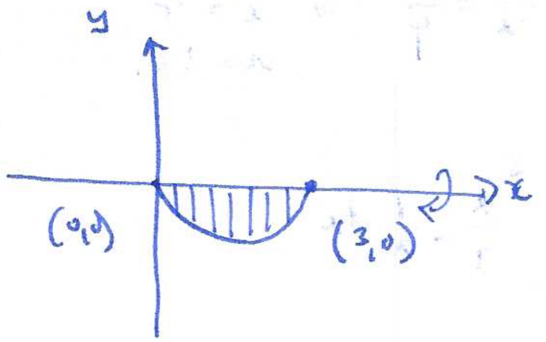
$$\int_1^2 \frac{1}{x} - \frac{x}{4}\,dx + \int_2^3 \frac{x}{4} - \frac{1}{x}\,dx$$

$$= \left[\ln|x| - \frac{x^2}{8}\right]_1^2 + \left[\frac{x^2}{8} - \ln|x|\right]_2^3$$

$$= \ln(2) - \frac{1}{2} - \ln(1) + \frac{1}{8} + \frac{9}{8} - \ln(3) - \frac{1}{2} + \ln(2)$$

$$= 2\ln(2) - \ln(3) + \frac{1}{4}$$

(3) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 3x$ and the $x$-axis. Find the volume of revolution of this region rotated about the $x$-axis.



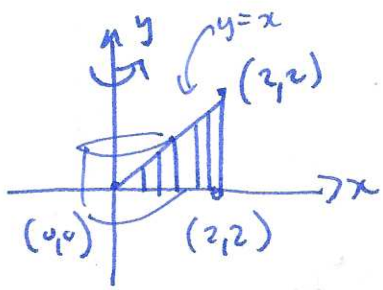

$$V = \int A(x)\,dx$$

$$= \int_0^3 \pi r^2\,dx = \int_0^3 \pi y^2\,dx$$

$$= \pi \int_0^3 (x^2-3x)^2\,dx = \pi \int_0^3 x^4 - 6x^3 + 9x^2\,dx$$

$$= \pi \left[ \frac{1}{5}x^5 - \frac{3\cancel{2}}{2}x^4 + 3x^3 \right]_0^3 = \pi \left( \frac{243}{5} - \frac{243}{2} + 81 \right) = \frac{81\pi}{10}$$

(4) (10 points) Find the volume of revolution of the triangle with vertices $(0,0)$, $(2,2)$ and $(2,0)$, rotated about the $y$-axis.




$r = x$

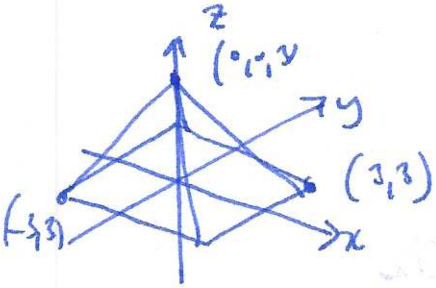
$h = y$

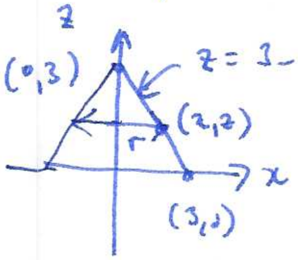
$$V = \int A(x)\,dx = \int_0^2 2\pi r h\,dx$$

$$= \int_0^2 2\pi x y\,dx = \int_0^2 2\pi x^2\,dx$$

$$= 2\pi \left[\frac{1}{3}x^3\right]_0^2 = \frac{16\pi}{3}$$

(5) (10 points) Find the volume of the pyramid whose base is the square in the $xy$-plane with vertices $(\pm 3, \pm 3, 0)$ and whose top point is $(0, 0, 3)$.




$r = 2x.$

$$V = \int A(z)\,dz = \int_0^3 r^2\,dz.$$

$$= \int_0^3 4x^2\,dz = \int_0^3 4\cdot(3-z)^2\,dz$$

$$= \left[\, 4\int_0^3 9 - 6z + z^2\,dz\right.$$

$$= 4\left[9z - 3z^2 + \frac{1}{3}z^3\right]_0^3 = 4(27 - 27 + 9) = 36$$

$$\int uv'\,dx = uv - \int u'v\,dx$$



(6) (10 points) Find $\int \underbrace{x}_{u}\underbrace{e^{-2x}}_{v'}\,dx$.

$u = x \quad u' = 1$

$v' = e^{-2x} \quad v = -\frac{1}{2}e^{-2x}$

$$\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \int 1 \cdot -\frac{1}{2}e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} \overset{-}{\cancel{+}} \frac{1}{4}e^{-2x} + c$$

$$\int uv'\,dx = uv - \int u'v\,dx$$



(7) (10 points) Find $\int_0^{\pi} \underbrace{e^{3x}}_{u} \underbrace{\cos(x)}_{v'}\, dx$.

$$\int_0^{\pi} e^{3x}\cos(x)\,dx = \underbrace{\left[e^{3x}\sin(x)\right]_0^{\pi}}_{=0} - \int_0^{\pi} \underbrace{3e^{3x}}_{u}\,\underbrace{\sin(x)}_{v'}\,dx$$

$$\int_0^{\pi} e^{3x}\cos(x)\,dx = -\left[3e^{3x}\cdot -\cos x\right]_0^{\pi} + \int_0^{\pi} 9e^{3x}\cdot -\cos x\,dx$$

$$10\int_0^{\pi} e^{3x}\cos(x)\,dx = -3e^{3\pi} - 3$$

$$\int_0^{\pi} e^{3x}\cos(x)\,dx = \frac{-3(e^{3\pi}+1)}{10}$$

(8) Find $\int \sin^3 x\, dx$. $= \int \sin x\,(1-\cos^2 x)\,dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$\int \sin x\,(1-u^2)\,\frac{dx}{du}\,du = \int \sin x\,(1-u^2)\,\frac{1}{-\sin x}\,dx = \int u^2 - 1\,du$

$= \frac{1}{3}u^3 - u + c = \frac{1}{3}\cos^3 x - \cos x + c$

(9) Find $\int \sin(5x)\cos(2x)\,dx$.

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2\sin(A)\cos(B)$

$\int \sin(5x)\cos(2x)\,dx = \frac{1}{2}\int \sin(7x) + \sin(3x)\,dx$

$= -\frac{1}{14}\cos(7x) - \frac{1}{6}\sin(3x) + c$

(10) Find $\int \frac{3x+1}{(x+1)^2(x-1)}\,dx$.

$$\frac{3x+1}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} = \frac{A(x+1)(x-1)+B(x-1)+C(x+1)^2}{(x+1)^2(x-1)}.$$

$x=1$ : $4 = 4C$ $\quad C = 1$

$x=-1$ : $-2 = -2B$ $\quad B = 1$

$x=0$ : $1 = -A - B + C$ $\quad A = -1$

$$\int \frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{1}{x-1}\,dx = -\ln|x+1| - \frac{1}{x+1} + \ln|x-1| + C$$