Math 232 Calculus 2 Spring 25 Sample final

1. Using Implicit differentiation derive the formula $\dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}$

2. Find the equation of tangent to the curve $y = \arccos(1/x^3)$ at the point $(1, 0)$.

3. Sketch the region enclosed by the curves $y = x^2$ and $y = x + 2$ and find its area.

4. Sketch the region enclosed by the curves $y = \sin 2x$ and $y = \cos x$ between $x = 0$ and $x = \pi/2$ and find its area.

5. Find the volume of the solid obtained by revolving the region bounded by the curves $y = x^2$ and $x = y^2$ about the line $x = -1$ using the discs method as well as the shell method.

6. Set up (but do not evaluate) integrals to find the volume of the solid obtained by revolving the region bounded between the curves $y = x^2 - 4$ and the line $y = -x + 2$ about the following axes:

   (a) $y = 8$ (use discs/washers)

   (b) $y = -4$ (use shells)

7. Find the volume of the given solid obtained by rotating the region bounded by given curves about the specified axis.

   (a) $y = x^2, y = 0, x = 4$ about x-axis.

   (b) $y = x^3, y = 6x - x^2$ in the first quadrant rotated about x-axis.

8. Find the arc length of the following curves.

   (a) $y = \ln(\cos x)$ from $x = 0$ to $x = \pi/3$.

   (b) $x = \frac{1}{2}(e^y + e^{-y})$ from $y = 0$ to $y = 3$.

9. Find the length of the curve $y = x^{3/2}$ for $0 \leq x \leq 4$.

10. Evaluate the following integrals.

(a) $\int e^{3x+4}\ dx$

(b) $\int \frac{1}{ax-b} dx$

(c) $\int_0^1 \frac{3t}{(t^2+5)^4} dx$

(d) $\int \sin(xy-z) dy$

(e) $\int_1^{e^2} \frac{2-\ln x}{x} dx$

(f) $\int xe^{4x} dx$

(g) $\int s^2 e^{2s} ds$

(h) $\int_1^3 \ln x\ dx$

(i) $\int t^4 \ln t\ dt$

(j) $\int \cos x e^x dx$

(k) $\int \cos^{-1} x\ dx$

(l) $\int \sin^2(x) dx$

(m) $\int_0^{\pi} \cos^2(x) dx$

(n) $\int \tan y\ dy$

(o) $\int \cos^3 x \sin^2 x\ dx$

(p) $\int \sec t\ dt$

(q) $\frac{1}{x^2-6x-16} dx$

(r) $\int \frac{7t+1}{t^2+t-6} dt$

(s) $\int \frac{1}{y^2-a^2} dy$

(t) $\int \frac{9t+1}{3t+4}\ dt$

(u) $\int \frac{x^3+4x-3}{x^2+4}\ dx$

(v) $\int \frac{3y^2+2}{y^2+4}\ dy$

(w) $\int \frac{3}{t^2+4t+5}\ dt$

(x) $\int \frac{3x+4}{x^2+9}\ dx$

(y) $\int \frac{2x-3}{x^2+10x+30}\ dx$

(z) $\int y+2\sqrt{2+3y} dy$

(aa) $\int \frac{x^2}{(x^2-2x+1)^2}\ dx$

(ab) $\int \frac{e^t}{e^{2t}+3e^t+2}\ dx$

(ac) $\int \frac{1}{\sqrt{y}(y-1)}\ dy$

(ad) $\int_1^{\infty} \frac{1}{(2x+1)^3}\ dx$

(ae) $\int_2^6 \frac{y}{\sqrt{y-2}}\ dy$

(af) $\int_{-\infty}^0 e^{2t}\ dt$

11. Determine whether the following series converge or diverge. Indicate which test you are using.

(a) $\sum_{n=1}^{\infty} \frac{3^n}{5^{n-1}}$

(b) $\sum_{n=1}^{\infty} 2^{1-n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} - \frac{3}{n^3}$

(d) $\sum_{n=1}^{\infty} \frac{1+2^n+3^n}{5^n}$

(e) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$

(f) $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

(g) $\sum_{n=1}^{\infty} \frac{n^2-n}{n^4+2}$

(h) $\sum_{n=1}^{\infty} \frac{n}{n^3-1}$

(i) $\sum_{n=1}^{\infty} \frac{n^2+1}{3n^2+2}$

(j) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$

(k) $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n+5}$

(l) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{3n+2}$

(m) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2^n+1}}$

(n) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^{3/2}}$

(o) $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

(p) $\sum_{n=1}^{\infty} \frac{n^3}{5^n}$ (q) $\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

12. Express the following repeating decimal as a fraction.

    (a) $1.0022222222\ldots$ (b) $3.4343434343\ldots$

13. Suppose that the series $\sum_{n=1}^{\infty} c_n(x+1)^n$ converges when $x = 2$ and diverges when $x = 4$. What can you say about the convergence or divergence of the following series?

    (a) $\sum_{n=1}^{\infty} c_n$

    (b) $\sum_{n=1}^{\infty} c_n(-1)^n$

    (c) $\sum_{n=1}^{\infty} c_n(-1)^n 7^n$

    (d) $\sum_{n=1}^{\infty} c_n(-1)^n 5^n$

14. Find power series representations of the following functions (You may choose the center).

    (a) $f(x) = \tan^{-1}(2x)$

    (b) $f(x) = \dfrac{x^4}{(1+x)^2}$

    (c) $f(x) = \ln(1+x)$

    (d) $f(x) = e^{(x-2)^2}$

    (e) $f(x) = \dfrac{\sin(2x^2)}{x^2}$

    (f) $f(x) = \int e^{-x^2}\,dx$

15. Find Taylor series of the given function at given point.

    (a) $f(x) = e^{2x}, a = 2$

    (b) $f(x) = 1/x, a = -3$

    (c) $f(x) = \sqrt{1+x}, a = 0$

16. By recognizing each of the following series as a Taylor series evaluated at a particular value of x, or otherwise, find the sum of each of the following convergent series.

    (a) $1 + \frac{2}{1!} + \frac{4}{2!} + \frac{8}{3!} + \cdots + \frac{2^n}{n!} + \cdots$

    (b) $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \cdots + \frac{(-1)^n}{(2n+1)!} + \cdots$

(c) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots + \frac{1}{4^n} + \cdots$

(d) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^{n+1}}{n} + \cdots$

1

17. Find the Taylor or Maclaurin polynomial for the given function, for the given degree, centered at the given point.

    (a) $f(x) = x^2, n = 3, c = 1$

    (b) $f(x) = \frac{1}{x^2}, n = 4, c = 1$

    (c) $f(x) = \ln x, n = 4, = 2$

    (d) $f(x) = \sin(3x), n = 5, c = 0$

    (e) $f(x) = \sqrt{x}, n = 4, c = 1$

18. How good are the following approximations? (use error bound for Taylor polynomial)

    (a) $\cos(0.3) \sim 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$

    (b) $e \sim 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}$

19. Find the equation of tangent at the given point on the given parametric curve.

    (a) $x = 3\cos t, y = \sin t$ at $t = 0$.

    (b) $x = \sqrt{t-1}, y = \sqrt{t}$ at $t = 2$.

20. Find all the points of horizontal and vertical tangency to the parametric curve $x = t^2 - t + 2$ and $y = t^3 - 3t$.

21. Find a parameterization for the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$, and use this to find:

    (a) The length of the curve.

    (b) The surface area of the shape obtained by rotating this curve around the x-axis.

22. Draw the curve in polar coordinates given by $r = 1 + \sin\theta$.

    (a) Find the area enclosed by the curve.

(b) Write down the formula for the length of the curve

(c) Find the arc length of the curve. (Hint: set $\theta = 2t$ and the observe that the expression in the square root is a perfect square.)