SF1

Q1

$y = \sin^{-1}(x)$

$\sin(y) = x$

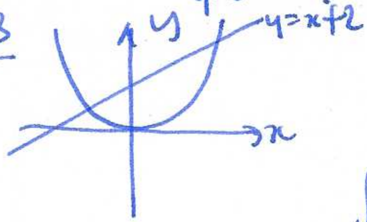
$\cos(y)\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$

Q2

$y = \cos^{-1}\left(\frac{1}{x^3}\right)$ $\quad \frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{x^3}\right)^2}} \cdot -3x^{-4}$ ← vertical at $(1,0)$ $x = 1$.

Q3



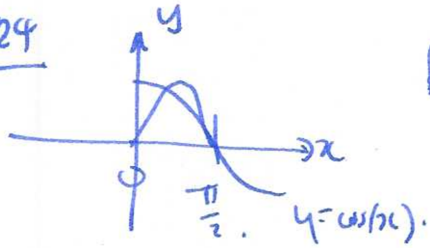
intersections: $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \quad x = -1, 2$.

$\int_{-1}^{2} x + 2 - x^2 \, dx = \left[\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3\right]_{-1}^{2} = 2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= \frac{36 - 8 - 3 + 12 - 2}{6} = \frac{35}{6}$

Q4



find intersections: $\sin 2x = \cos x$

$2\sin x \cos x = \cos x$

$\cos x\,(2\sin x - 1) = 0$

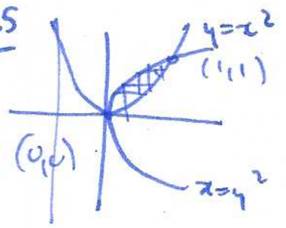
$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$.

$\sin x = \frac{1}{2} \quad x = \pi/6$

$\int_0^{\pi/6} \cos x - \sin 2x \, dx + \int_{\pi/6}^{\pi/2} \cancel{\cos x}\ \sin 2x - \cos x \, dx$

$= \left[+\sin x + \frac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\frac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2} = +\frac{1}{2} + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - 1 + \frac{1}{4} + \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{\sqrt{3}}{2}$

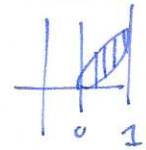
Q5



discs:

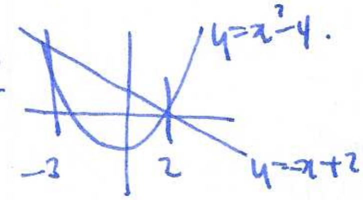
$\pi\int_0^1 (1+\sqrt{y})^2 - (1+y^2)^2 \, dy$

shells.



$2\pi\int_0^1 (1+x)(x^2 - \sqrt{x})\,dx$

Q6



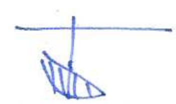
intersections: $x^2 - 4 = -x + 2$.

$x^2 + x - 6 = 0$

~~x = −1±√1+~~

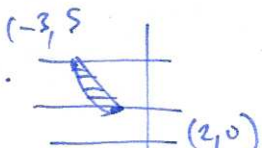
$(x+3)(x-2) = 0$

a) $y = 8$ discs.



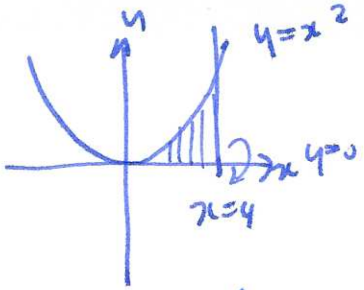
$\pi\int_{-3}^{2} \left(8 - (x^2 - 4)\right)^2 - \left(8 - (-x+4)\right)^2 dx.$

b) $y = -4$ shells.



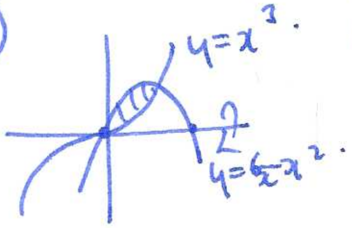
$2\pi\int_0^5 (4+y)\left(\frac{1}{-2-y-\sqrt{4+y}}\right) dy$

Q7 a)



$\int_0^4 \pi x^4 \, dx$

b)



intersection $x^3 = 6x - x^2$

$x(x^2+x-6)=0$

$x(x+3)(x-2)$.

$\pi \int_0^2 \cancel{x^6} \, (6x-x^2)^2 - x^6 \, dx$.

Q8 a) $y = \ln(\cos x)$ $\quad \int_0^{\pi/3} \sqrt{1+(f'(x))^2}\, dx$ $\quad y' = \frac{1}{\cos x} \cdot -\sin x$

$\int_0^{\pi/3} \sqrt{1+\tan^2 x}\, dx = \int_0^{\pi/3} \sec\theta \, d\theta = \Big[\ln|\sec\theta + \tan\theta| \Big]_0^{\pi/3}$.

b) $x = \frac{1}{2}(e^y + e^{-y})$ $\quad \int_0^3 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_0^3 \sqrt{1 + \frac{1}{4}e^{2y} - \frac{1}{2} + \frac{1}{4}e^{-2y}}\, dy$

$\frac{dx}{dy} = \frac{1}{2}e^y - \frac{1}{2}e^{-y}$

$= \int_0^3 \sqrt{\left(\frac{1}{2}e^y + \frac{1}{2}e^{-y}\right)^2}\, dy = \int_0^3 \frac{1}{2}e^y + \frac{1}{2}e^{-y}\, dy = \left[\frac{1}{2}e^y - \frac{1}{2}e^{-y}\right]_0^3$.

Q9 $y = x^{3/2}$ $\quad \frac{dy}{dx} = \frac{3}{2}x^{1/2}$ $\quad \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$. $\quad u = 1+\frac{9}{4}x \quad \frac{du}{dx} = \frac{9}{4}$.

$\int_1^{10} u^{1/2} \frac{dx}{du}\, du = \int_1^{10} \frac{4}{9} u^{1/2}\, du = \left[\frac{2}{9} u^{3/2}\right]_1^{10}$.

Q12 a) $1 + \frac{2}{1000} + \frac{2}{10^4} + \frac{2}{10^5} + \cdots = 1 + \frac{2/10^3}{1-10} = 1 + \frac{2}{9.10^3}$.

b) $3 + \frac{43}{10^2} + \frac{43}{10^4} + \cdots = 3 + \frac{43/10^2}{1 - 1/10^2} = 3 + \frac{43}{99}$.

Q13 $\sum_{n=1}^{\infty} c_n (x+1)^n$ converges for $x=2$, diverges for $x=4$ $\Rightarrow 3 \leq R \leq 5$.

center $x=-1$

a) converges b) converges c) diverges d) don't know.

Q14 a) $f(x) = \tan^{-1}(2x) = x$ $\qquad x - \frac{4}{3}x^3 + \frac{16}{5}x^5 \dots$

$f'(x) = \frac{1}{1+4x^2} = \cancel{1 - \frac{1}{2x^2} + \frac{1}{4x^4}} \dots + \dots$ $1 - 4x^2 + 16x^4 - \dots$

b) $f(x) = \frac{x^4}{(1+x)^2} = x^4 \cdot \frac{1}{(1+x)^2}$ $\qquad \int \frac{1}{(1+x)^2}\,dx = \frac{-1}{1+x} = -\left(1 - x + x^2 - x^3 + - \right)$

$= x^4\left(1 - 2x + 3x^2 - \dots\right).$

c) $f(x) = \ln(1+x) \ . = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$

$f'(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3$

d) $e^{(x-2)^2}$ center $x = 2$. $=$ $\qquad e^x = 1 + x + \frac{x^2}{2!} + \dots$

$e^{(x-2)^2} = 1 + (x-2)^2 + \frac{(x-2)^4}{2!} + \dots$

e) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$ $\qquad \frac{\sin(2x^2)}{x^2} = 2 - \frac{8x^4}{3!} + \frac{2^5 x^8}{5!} \dots$

f) $e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \dots$

$\int e^{-x^2}\,dx = x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7 \cdot 3!} + \dots$

Q15 a) $f(x) = e^{2x}$ $\quad a = 2$ $\qquad e^{2x} = e^{2((x-a)+a)} = e^{2a} \cdot e^{2(x-a)}.$

$= e^{2a}\left(1 + 2(x-a) + \frac{4(x-a)^2}{2!} + \dots\right).$

b) $f(x) = \frac{1}{x}$ $\quad a = -3,$ $\qquad f(x) = \frac{1}{(x+3)-3} = -\frac{1}{3}\,\frac{1}{1-(x+3)} = -\frac{1}{3}\left(1 - (x+3) + (x+3)^2 - (x+3)^3 + \dots\right)$

c) $f(x) = \sqrt{1+x}$; $a = 0$ $\qquad (1+x)^{1/2} = 1 + \frac{1}{2}x + \frac{1}{2}\left(\frac{1}{2} - 1\right)\frac{x^2}{2!} + \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)\frac{x^3}{3!} + \dots$

Q16 a) $e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\qquad e^2$.

b) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\qquad \sin(1)$.

c) $1/(1 - 1/4)$ $\qquad$ d) $\frac{1}{1-x} = 1 + x + x^2 + \dots$ $\qquad -\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$

$x = -1$, $\ln(1/2)$.

Q17 a) $f(x) = x^2$ $\quad f(1) = 1$

$f'(x) = 2x$ $\quad f'(1) = 2$

$f''(x) = 2$ $\quad f''(1) = 2$

$f^{(3)}(x) = 0$ $\quad f^{(3)}(1) = 0$

$$1 + 2(x-1) + 2\frac{(x-1)^2}{2!} + 0$$

b) $f(x) = x^{-2}$ $\quad f(1) = 1$

$f'(x) = -2x^{-3}$ $\quad f'(1) = -2$

$f''(x) = 6x^{-4}$ $\quad f''(1) = 6$

$f^{(3)}(x) = -24x^{-5}$ $\quad f^{(3)}(1) = -24$

$f^{(4)}(x) = +120x^{-6}$ $\quad f^{(4)}(1) = 120$

$$T_4 = 1 - 2(x-1) + 6\frac{(x-1)^2}{2!} - 24\frac{(x-1)^3}{3!} + 120\frac{(x-1)^4}{4!}$$

c) $f(x) = \ln(x)$ $\quad f(2) = \ln(2)$

$f'(x) = x^{-1}$ $\quad f'(2) = 1/2$

$f''(x) = -x^{-2}$ $\quad f''(2) = -1/4$

$f^{(3)}(x) = 2x^{-3}$ $\quad f^{(3)}(2) = 1/4$

$f^{(4)}(x) = -6x^{-4}$ $\quad f^{(4)}(2) = -6/16 = -3/8$

$$T_4 = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{4}\frac{(x-2)^2}{2!} + \frac{1}{4}\frac{(x-2)^3}{3!} - \frac{3}{8}\frac{(x-2)^4}{4!}$$

d) $f(x) = \sin(3x)$ $\quad f(0) = 0$

$f'(x) = 3\cos(3x)$ $\quad f'(0) = 3$

$f''(x) = -9\sin(3x)$ $\quad f''(0) = 0$

$f^{(3)}(x) = -27\cos(3x)$ $\quad f^{(3)}(0) = -27$

$f^{(4)}(x) = 81\sin(3x)$ $\quad f^{(4)}(0) = 0$

$f^{(5)}(x) = 243\cos(3x)$ $\quad f^{(5)}(0) = 243$.

$$T_5 = 3x - \frac{27x^3}{3!} + \frac{243x^5}{5!}$$

e) $f(x) = x^{1/2}$. $\quad f(1) = 1$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(1) = 1/2$

$f''(x) = -\frac{1}{4}x^{-3/2}$ $\quad f''(1) = -1/4$

$f^{(3)}(x) = \frac{3}{8}x^{-5/2}$ $\quad f^{(3)}(1) = 3/8$

$f^{(4)}(x) = \frac{-15}{16}x^{-7/2}$ $\quad f^{(4)}(1) = \frac{-15}{16}$

$$T_4 = 1 + \frac{1}{2}(x-1) - \frac{1}{4}\frac{(x-1)^2}{2!} + \frac{3}{8}\frac{(x-1)^3}{3!} - \frac{15}{16}\frac{(x-1)^4}{4!}$$

Q18 a) $\cos(0.3) \sim 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$

error bound $\leq \frac{K(0.3)^5}{5!}$ can choose $K=1$.

b) $e = 1+1+\frac{1}{2!}+\dots\frac{1}{5!}$ error bound $\leq \frac{K(1)^6}{6!}$ can choose $K=3$.

Q19 a) $c(t) = (3\cos t, \sin t)$ $c'(t) = (-3\sin t, \cos t)$ $c'(0) = (0,1)$ $\frac{dy}{dx} = 1/0$

$x = 1$.

b) $c(t) = (\sqrt{t-1}, \sqrt{t})$ $c'(t) = (\frac{1}{2}(t-1)^{-1/2}, \frac{1}{2}t^{-1/2})$ $c'(2) = (\frac{1}{2}, \frac{1}{2\sqrt{2}})$ $c(2) = (1, \sqrt{2})$

$\frac{dy}{dx} = \frac{1/2\sqrt{2}}{1/2} = \frac{1}{\sqrt{2}}$. $y - \sqrt{2} = \frac{1}{\sqrt{2}}(x-1)$.

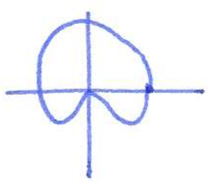
Q20 $c(t) = (t^2-t+2, t^3-3t)$ $c'(t) = (2t-1, 3t^2-3)$.

vertical: $2t-1$ $t=\frac{1}{2}$ horizontal: $3t^2-3=0$ $t=\pm 1$.

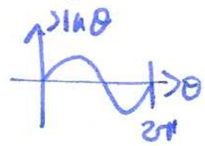
Q21 $c(t) = (t, 2t^2)$ $c'(t) = (1, 4t)$.

arc length $\int_0^1 \sqrt{1+16t^2}\, dt$ surface area $2\pi\int_0^1 2t^2\sqrt{1+16t^2}\, dt$.

Q22



$r = 1 + \sin\theta$.



area $= \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2\, d\theta$

length $= \int_0^{2\pi} \sqrt{(1+\sin\theta)^2 + \cos^2\theta}\, d\theta = \int_0^{2\pi} \sqrt{2+2\sin\theta}\, d\theta = \sqrt{2}\int_0^{2\pi} \sqrt{1+\sin\theta}\, d\theta$

$1+\sin 2t$.

$1+2\sin t\cos t$

$(\sin t+\cos t)^2$.

$= \sqrt{2}\int_0^{2\pi} \sin\frac{\theta}{2} + \cos\frac{\theta}{2}\, d\theta$.