MTH 306 History of Math, Fall 2025

For Wed Nov 12th

(1) Read handout 11.

HW9 Due Wed Nov 17th

(1) For $a \in \mathbb{C}$ let h_a denote a half turn about a. For any straight line l, let r_l denote reflection in l.
   (a) What is $r_l r_m$ if (i) l and m intersect, or (ii), l and m are parallel?
   (b) Show that $r_l r_m = r_m r_l$ if and only if l and m are equal or orthogonal.
   (c) Show that $h_a r_l = r_l h_a$ if and only if a lies on l.
   (d) Show that $h_a h_b = h_c h_a$ if and only if a is the midpoint of bc.
   (e) Show that $h_a r_l = r_l h_b$ if and only if l is the perpendicular bisector of bc.
   (f) Under what circumstances does $h_a h_b h_c h_d = 1$?
   (g) Under what circumstance does $r_l r_m = r_n r_l$?

(2) A Möbius map is a map $f\colon \mathbb{C}\cup\{\infty\} \to \mathbb{C}\cup\{\infty\}$ of the form $z \mapsto \frac{az+b}{cz+d}$, where $ad - bc \neq 0$.
   (a) Show that every Möbius map is the composition of maps of the form $z \mapsto 1/z$, $z \mapsto z + a$ and $z \mapsto az$, $a \in \mathbb{C}$.
   (b) Show that Möbius maps take circle and straight lines to circles and straight lines, not necessarily respectively.
   (c) Let (a, b, c) be an ordered triple of distinct points in $\mathbb{C} \cup \{\infty\}$. Show that there is a unique Möbius map sending $a \mapsto \infty$, $b \mapsto 0$ and $c \mapsto 1$. Deduce that fro any two ordered triples of distinct points, there is a map taking the first triple to the second.

(3) (a) Let γ_1 and γ_2 be two circles or straight lines described by the equations

$$A_i|z|^2 + B_i z + \overline{B_i}\overline{z} + C_i = 0,$$

   for $i = 1, 2$, where $A_i, C_i \in \mathbb{R}$ and $|B_i|^2 > A_i C_i$. Show that γ_1 and γ_2 cross orthogonally if and only if

$$B_1\overline{B_2} + \overline{B_1}B_2 = A_1C_2 + A_2C_1.$$

   (b) Use 1(a) and 2(a) to show that Möbius maps take orthogonal circles and straight lines to orthogonal circles and straight lines.