MTH 306 History of Math, Fall 2025

For Mon Nov 3rd

(1) Read handout 10.

HW8 Due Wed Nov 12th

(1) Determine the geometric effect on the complex plane of the following transformations.
  (a) $f(z) = z + a$, where $a \in \mathbb{C}$.
  (b) $f(z) = e^{i\theta} z$, where $\theta \in [0, 2\pi)$.
  (c) $f(z) = \overline{z}$.
  (d) $f(z) = \overline{z} + a$ for $a \in \mathbb{R}$.

(2) (a) Show that every isometry of $\mathbb{R}^2$ may be written in the form $z \mapsto e^{i\theta} z + a$ or $z \mapsto e^{i\theta}\overline{z} + a$, for $\theta \in [0, 2\pi)$ and $a \in \mathbb{C}$.
  (b) Classify isometries in terms of θ and a.

(3) For each of the following pairs of transformations, calculate f^{-1}, $f \circ g$ and $f \circ g \circ f^{-1}$, and descibe their geometric effects.
  (a) $f(z) = e^{i\theta} z$, $g(z) = z + a$, where $\theta \in (0, 2\pi)$ and $a \in \mathbb{C} \setminus \{0\}$.
  (b) $f(z) = z + a$, $g(z) = e^{i\theta}$, where $\theta \in (0, 2\pi)$ and $a \in \mathbb{C} \setminus \{0\}$.
  (c) $f(z) = e^{i\theta}$, $g(z) = \overline{z}$, where $\theta \in (0, 2\pi)$.
  (d) $f(z) = \overline{z}$, $g(z) = e^{i\theta}$, where $\theta \in (0, 2\pi)$.

(4) (a) Show that the general equation of a straight line in $\mathbb{C}$ is given by $Bz + \overline{B}\overline{z} + C = 0$, where $B \in \mathbb{C} \setminus \{0\}$ and $C \in \mathbb{R}$.
  (b) Show that the general equation of a circle in $\mathbb{C}$ is given by $A|z|^2 + Bz + \overline{B}\overline{z} + C = 0$, where $A, C \in \mathbb{R}$, $B \in \mathbb{C}$, $A \neq 0$ and $|B|^2 > AC$. Where does the condition $|B|^2 > AC$ come from?
  [Hint: in (a) write the equation for a straight line in $\mathbb{R}^2$, and use $x = (z + \overline{z})/2$ and $y = i(\overline{z} - z)/2$. In (b) use $(z - c)(\overline{z} - \overline{c}) = |z - c|^2$ to define the equation of a circle of center c.]

(5) Let $A \subseteq \mathbb{R}^2$. The subgroup stab(A) consists of all isometries f such that $f(A) = A$.
  (a) Explicitly identify stab(0) and stab$(\mathbb{R})$.
  (b) Use conjugation to identify stab(w) and stab(L), for w a point and L a line.
  (c) Explicitly identify stab$(\{0, 1\})$. Is it equal to stab$(0) \cap$ stab(1)?