

## MTH 306 History of Math, Fall 2025

**For Wed Nov 19th**

- (1) Read handout 12.

**HW10 Due Wed Nov 26th**

- (1) Construct explicit Turing machines which do the logical operations of AND and OR.
- (2)
  - (a) The group  $\text{PSL}(2, \mathbb{Z})$  has a presentation  $\langle a, b \mid a^2, b^3 \rangle$ . Find elements of order 2 and 3 in  $\text{PSL}(2, \mathbb{Z})$ . Can they be conjugate?
  - (b) Can you characterize the elements of order 2? Hint: think about eigenvectors / traces.
  - (c) Show that  $\text{PSL}(2, \mathbb{Z})$  is generated by the pair of matrices  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ . Are these elements conjugate?
  - (d) Find a presentation for  $\text{PSL}(2, \mathbb{Z})$  with the generators from (c).
  - (e) Are all elements with the same trace conjugate?
- (3) Show that the following group has trivial abelianization.

$$\langle a, b \mid a^3 b^3 a^{-2} b^{-2}, a^{-3} b^{-1} a^5 b^2 \rangle$$

Hard: Can you show that the group is the trivial group?