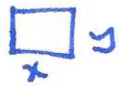
t Optimization

Example a piece of wire of length L is bent into a rectangle. What is the largest possible area?



area $A = xy$
length $L = 2x+2y$ $\Big\}$ $y = \frac{L}{2} - x$

$$A = x\left(\frac{L}{2} - x\right) = \frac{L}{2}x - x^2$$

$A'(x) = \frac{L}{2} - 2x$ critical point $x = \frac{L}{4}$ (local max), so max occurs when $x = y = \frac{L}{4}$ and area is $\frac{L^2}{16}$.

Example what shape of cylindrical can minimizes surface area, if you want total volume to be $1 ft^3$?

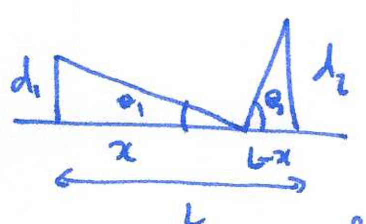
$V = \pi r^2 h = 1 \Rightarrow h = \frac{1}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + \frac{2\pi r \cdot 1}{\pi r^2} = 2\pi r^2 + \frac{2}{r}$

$A'(r) = 4\pi r - \frac{2}{r^2}$ solve $A'(r) = 0$: $r^3 = \frac{1}{2\pi}$ $r = \sqrt[3]{1/2\pi}$

Example a ball bounces off a wall. Show that the path which minimizes distance has equal angles of incidence and reflection.



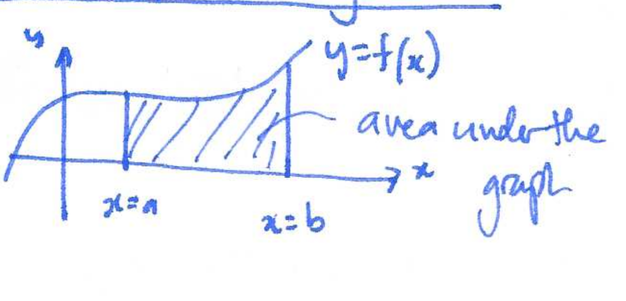
$f(x) = \text{distance} = \sqrt{d_1^2 + x^2} + \sqrt{d_2^2 + (L-x)^2}$

$f'(x) = \frac{1}{2}(d_1 + x^2)^{-1/2} \cdot 2x + \frac{1}{2}(d_2^2 + (L-x)^2)^{-1/2} \cdot 2(L-x) \cdot (-1)$

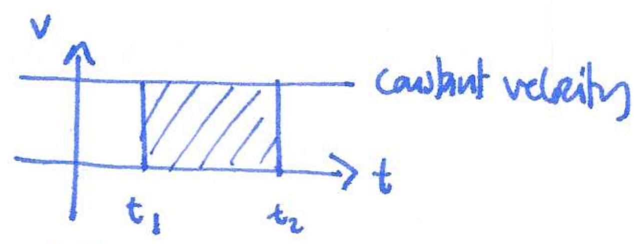
$f'(x) = 0$: $\frac{x}{\sqrt{d_1^2 + x^2}} = \frac{L-x}{\sqrt{d_2^2 + (L-x)^2}} \Leftrightarrow \cos\theta_1 = \cos\theta_2$

$\Rightarrow \theta_1 = \theta_2$

§5.1 Approximating area



Example



distance travelled = velocity × time
= area under the graph