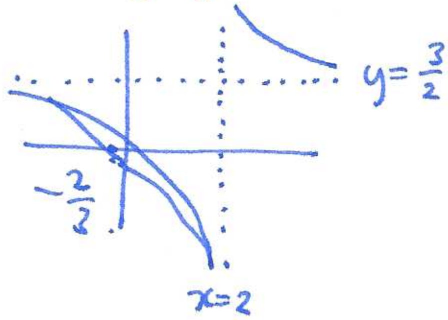
(3) f decreasing, no critical points except at vertical asymptote $x = 2$

(3) $f''(x) = \dfrac{8}{(x-2)^3}$ +ve $x > 2$ concave up
−ve $x < 2$ concave down



(4) horizontal asymptotes $\lim\limits_{x\to\infty} \dfrac{3x+2}{2x-4} \overset{l'H}{=} \lim\limits_{x\to\infty} \dfrac{3}{2} = \dfrac{3}{2}$

behaviour near asymptote/zero

$3x+2$	−		+		+
$2x-4$	−		−		+
		$-2/3$		2	
$f(x)$	+		−		+

<u>Example</u> sketch graph of $f(x) = \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-1/2}$

(1) vertical asymptotes: none

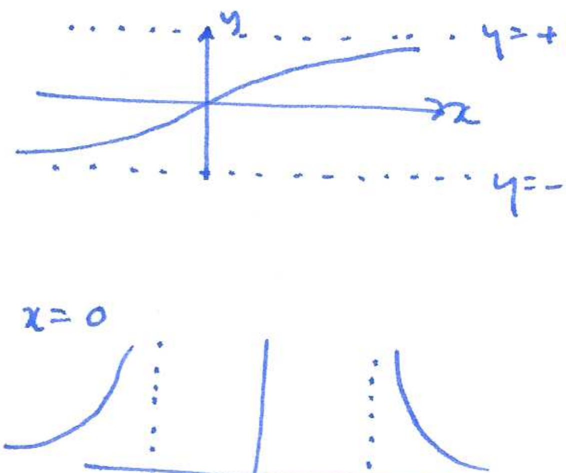
(2) find $f'(x) = \dfrac{\sqrt{x^2+1}\cdot 1 - x\cdot\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}{x^2+1} = \dfrac{x^2+1-x^2}{(x^2+1)^{3/2}} = (x^2+1)^{-3/2} > 0$

$\Rightarrow$ f increasing, no critical points.

(3) $f''(x) = -\frac{3}{2}(x^2+1)^{-5/2}\cdot 2x$ inflection point at $x=0$ +ve for $x < 0$
−ve for $x > 0$

(4) horizontal asymptotes $\lim\limits_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}} \overset{l'H}{=} \lim\limits_{x\to\infty} \dfrac{1}{\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x} = \lim\limits_{x\to\infty} \sqrt{1+1/x^2} = 1$

$\lim\limits_{x\to-\infty} \dfrac{x}{\sqrt{x^2+1}} = \lim\limits_{x\to+\infty} \dfrac{-x}{\sqrt{x^2+1}} = -1$

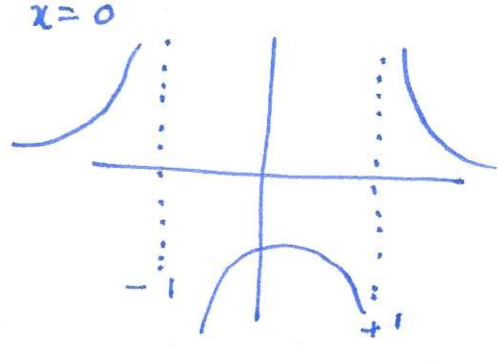


<u>Example</u> $f(x) = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

(1) vertical asymptotes at $x = \pm 1$

(2) $f'(x) = -(x^2-1)^{-2}\cdot 2x$ critical point at $x=0$

(3) $f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$



(4) etc.