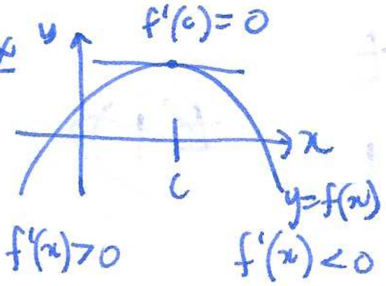
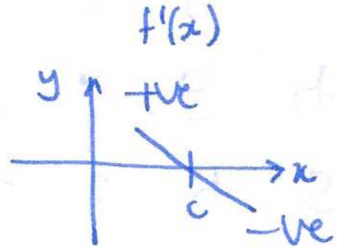
② $f(x) = x^2 - 2x - 3$ $\qquad$ $f'(x)$ where $2x - 2 > 0$

$f'(x) = 2x - 2$ $\qquad\qquad\qquad\qquad$ $x > 1$

First derivative test

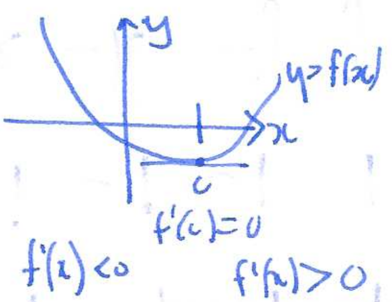
local max

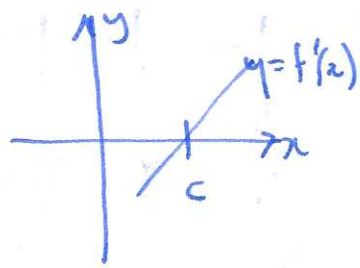




if $f'(x)$ goes from positive to negative at c $\Rightarrow$ c is local max

local min





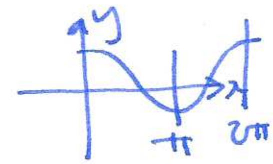
if $f'(x)$ goes from negative to positive at c $\Rightarrow$ c is local min.

Thm (First derivative test) If $f(x)$ is differentiable and $f'(c) = 0$ then if $f'(x)$ changes from +ve to −ve at c $\Rightarrow$ c local max

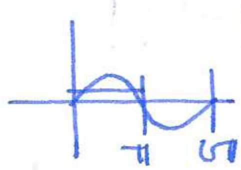
−ve to +ve $\Rightarrow$ c local min $\square$

Example classify critical points of $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$

- find critical points: $f'(x) = -2\cos(x)\sin x + \cos x$
- solve $f'(x) = 0$ $\qquad$ $\cos(x)(1 - 2\sin x) = 0$ $\qquad$ • $\cos(x) = 0$, $x = \frac{\pi}{2}$



$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ so critical points are $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

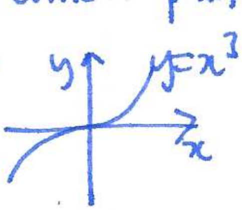
find sign of $f'(x)$:

	0 – $\pi/6$	$\pi/6$ – $\pi/2$	$\pi/2$ – $5\pi/6$	$5\pi/6$ – π
$\cos x$	+	+	−	−
$1 - 2\sin x$	+	−	−	+
$f'(x)$	+	−	+	−

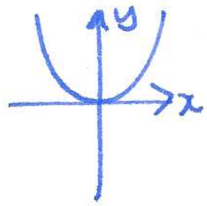
at $\frac{\pi}{6}$: local max; at $\frac{\pi}{2}$: local min; at $\frac{5\pi}{6}$: local max

Example critical point not max or min

$f(x) = x^3$



$f'(x) = 3x^2$



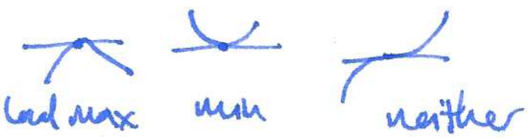
$f'(x) = 0$ at $x = 0$

$f'(x)$ +ve 0 +ve

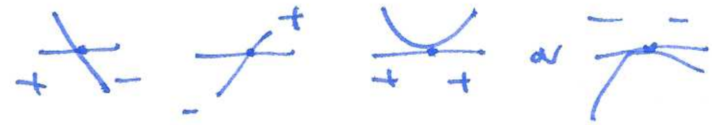
$\Rightarrow x = 0$ not local max or min.

recall critical point $f'(x) = 0$

$f(x)$



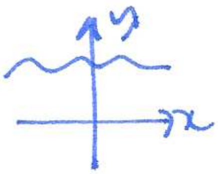
$f'(x)$



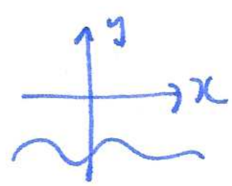
$f''(x)$ -ve +ve 0 0

§4.4 second derivative test

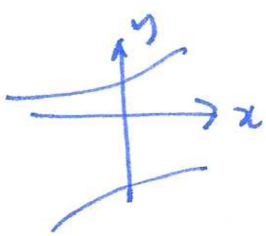
recall $f(x) > 0$
f positive



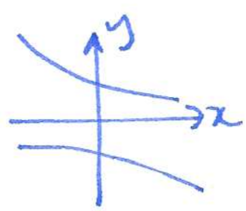
$f(x) < 0$
f negative



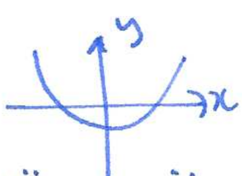
$f'(x) > 0$
f increasing



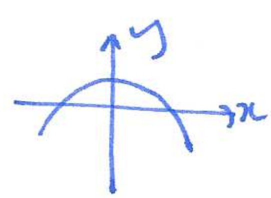
$f'(x) < 0$
f decreasing



$f''(x) > 0$
f concave up



$f''(x) < 0$
f concave down



mnemonic

Q: how do the slopes change?

concave up $\Leftrightarrow$ slopes increasing

$\Leftrightarrow f''(x) > 0$

concave down $\Leftrightarrow$ slopes decreasing

$\Leftrightarrow f''(x) < 0$

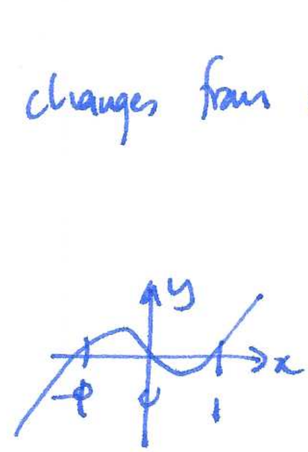
Defn Let $f(x)$ be differentiable on an interval (a,b), then

$f(x)$ is concave up $\Leftrightarrow f''(x) > 0$

$f(x)$ is concave down $\Leftrightarrow f''(x) < 0$

Def: An inflection point is a point where the graph changes from concave up to concave down, or vice versa.

Note: inflection point $\Rightarrow f''(x) = 0$, $\not\Leftarrow$



Example $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$

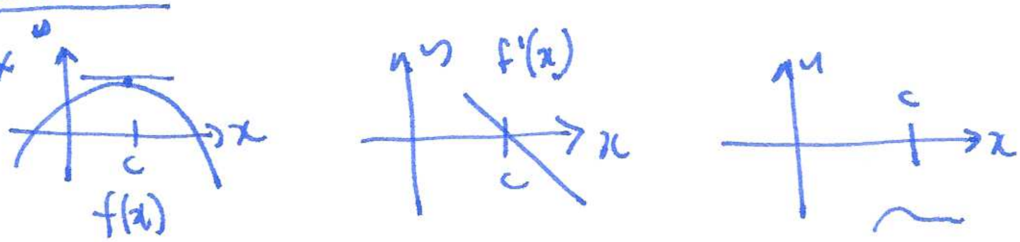
$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

$\left.\begin{array}{l} f''(x) > 0 \text{ for } x > 0 \\ f''(x) < 0 \text{ for } x < 0 \end{array}\right\}$ $x = 0$ is an inflection point.

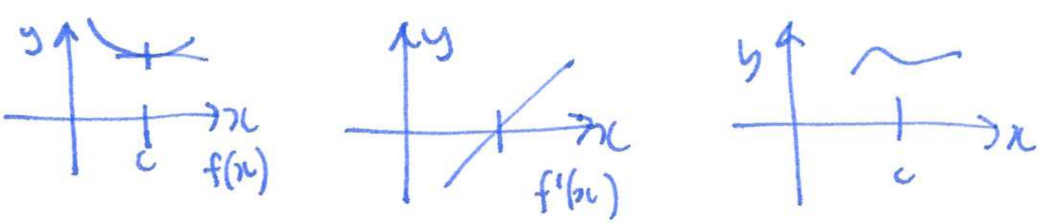
second derivative

Local max



$f''(c) < 0$

Local min



$f''(c) > 0$

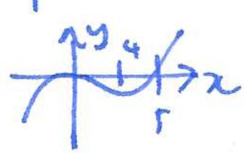
Thm Suppose $f(x)$ is differentiable and c is a critical point,

if $f''(c) > 0 \Rightarrow c$ is local min

$f''(c) < 0 \Rightarrow c$ is local max

$f''(c) = 0$ NO INFORMATION (may be local max/min/neither)

Example $f(x) = x^5 - 5x^4 = x^4(x-5)$



$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$

$f''(x) = 20x^3 - 60x^2$

critical points $f'(x) = 0$, $x = 0, 4$

2nd derivative test: $x = 0$ $f''(0) = 0$ no information, $x = 4$ $f''(4) = 320 > 0$ local min