Math 231 Calculus 1 Fall 25 Midterm 3b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a US Letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x\to 0} \dfrac{\sin(4x)}{e^{-3x}-1}$.

$\overset{l'H}{=} \lim_{x\to 0} \dfrac{4\cos(4x)}{-3e^{-3x}} = -\dfrac{4}{3}$

(2) Find $\lim_{x\to 0} \dfrac{\sqrt{1-2x^2}-1}{\cos(3x)-1}$.

$\overset{L'H}{=} \lim_{x\to 0} \dfrac{\frac{1}{2}(1-2x^2)^{-1/2}\cdot(-4x)}{-3\sin(3x)}$

$= \underbrace{\lim_{x\to 0} \dfrac{2}{\sqrt{1-2x^2}}}_{=2} \cdot \underbrace{\lim_{x\to 0} \dfrac{x}{3\sin(3x)}}_{\overset{L'H}{=} \lim_{x\to 0} \frac{1}{9\cos(3x)} = \frac{1}{9}}$

$= \dfrac{2}{9}$

(3) Consider the function $f(x) = e^{-x} + 3x$ ~~for $x > 0$.~~

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = -e^{-x} + 3$ solve $f'(x) = 0$:

$-e^{-x} + 3 = 0$

$e^{-x} = 3$

$-x = \ln(3)$ $\quad x = -\ln(3)$

b) $f''(x) = e^{-x}$

$f''(-\ln(3)) = e^{\ln(3)} = 3 > 0$ local min.

(4) Consider the function $f(x) = \dfrac{x^4 - 3}{x}$. $= x^3 - \dfrac{3}{x}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptotes at $x = 0$, $\lim_{x \to \pm\infty} f(x) = \pm\infty$ no horizontal asymptotes.

b) $f'(x) = 3x^2 + \dfrac{3}{x^2}$

$f''(x) = 6x - \dfrac{6}{x^3}$ solve $f''(x) = 0$: $x = \dfrac{1}{x^3}$ $x^4 = 1$ $x = \pm 1$

c) $f''(x) = 6\left(\dfrac{x^4 - 1}{x^3}\right)$

	-1		0		1	
$x^4 - 1$	$+$	$-$		$-$		$+$
x^3	$-$		$-$		$+$	$+$
$f''(x)$	$-$	$+$		$-$		$+$

concave up $(-1, 0) \cup (1, \infty)$

down $(-\infty, -1) \cup (0, 1)$

(5) Find the area under the graph of $y = 3x^2 + 2$ between $x = -1$ and $x = 3$.

$$\int_{-1}^{3} 3x^2+2\,dx = \left[x^3+2x\right]_{-1}^{3} = 27+6-(-1-2) = 36$$

(6) Find the indefinite integral $\int 4\sin(x) - \frac{3}{x} + \frac{2}{\sqrt{x}}\, dx$.

$-4\cos(x) - 3\ln(x) + 4x^{1/2} + C$

(7) Find the indefinite integral $\int x \sin(3x^2 - 2)\, dx$.

$u = 3x^2 - 2$

$\frac{du}{dx} = 6x$

$$\int x \sin(u) \frac{dx}{du}\, du$$

$$\int x \sin(u) \frac{1}{6x}\, du = \frac{1}{6} \int \sin(u)\, du$$

$$= -\frac{1}{6} \cos(u) + c$$

$$= -\frac{1}{6} \cos(3x^2 - 2) + c$$

(8) Find the definite integral $\int_0^1 e^x\sqrt{4e^x+1}\,dx$.

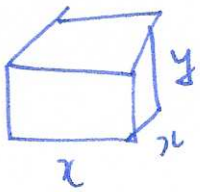
$u = 4e^x+1$

$\frac{du}{dx} = 4e^x$

$$\int_5^{4e+1} e^x u^{1/2}\frac{dx}{du}\,du = \int_5^{4e+1}\frac{1}{4}u^{1/2}\,du$$

$$= \left[\frac{1}{6}u^{3/2}\right]_5^{4e+1} = \frac{1}{6}\left((4e+1)^{3/2} - 5^{3/2}\right) \approx 4.9552\cdots$$

(9) A cardboard box has a square base and no top. What dimensions minimize surface area if the total volume of the box is 4m^3?



$V = x^2 y = 4$ $\quad y = \frac{4}{x^2}$

$A = x^2 + 4xy$

$A = x^2 + \frac{16}{x}$

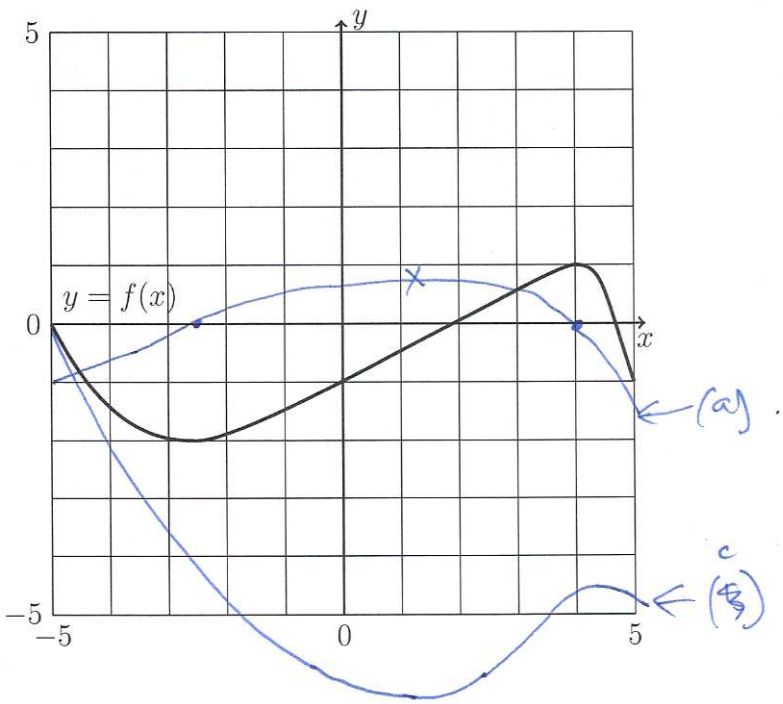
$\frac{dA}{dx} = 2x - \frac{16}{x^2}$ $\quad$ critical point: $\frac{dA}{dx} = 0$

$2x - \frac{16}{x^2} = 0$

$x^3 = 8 \quad x = 2$

$y = 1$.

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$.

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.