Math 231 Calculus 1 Fall 25 Midterm 3a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a US Letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x\to 0} \dfrac{1-e^{-3x}}{\sin(2x)}$. $\overset{l'H}{=} \lim_{x\to 0} \dfrac{3e^{-3x}}{2\cos(2x)} = \dfrac{3}{2}$

(2) Find $\lim_{x\to 0} \dfrac{\cos(4x)-1}{1-\sqrt{1-3x^2}}$.

$$\overset{\text{l'H}}{=} \lim_{x\to 0} \frac{-\sin(4x)\cdot 4}{-\frac{1}{2}(1-3x^2)^{-1/2}\cdot -6x}$$

$$= \lim_{x\to 0} -\frac{8}{6}\,\frac{4\sin(4x)\cdot\sqrt{1-3x^2}}{x}$$

$$\overset{\text{l'H}}{=} \lim_{x\to 0} -\frac{8}{6}\,\frac{\cos(4x)\cdot 4\cdot\sqrt{1-3x^2} + \sin(4x)\,\frac{1}{2}(1-3x^2)^{-1/2}\cdot(-6x)}{1} = -\frac{32}{6} = -\frac{16}{3}$$

(3) Consider the function $f(x) = e^{-x} + 2x$ ~~for $x > 0$~~.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = -e^{-x} + 2$ critical pt $f'(x) = 0$: $e^{-x} = 2$

$-x = \ln(2)$.

$x = -\ln(2)$

b) $f''(x) = e^{-x}$

$f''(-\ln(2)) = e^{\ln(2)} = 2 > 0$ local min

(4) Consider the function $f(x) = \dfrac{3 - x^4}{x}$. $= \dfrac{3}{x} - x^3$

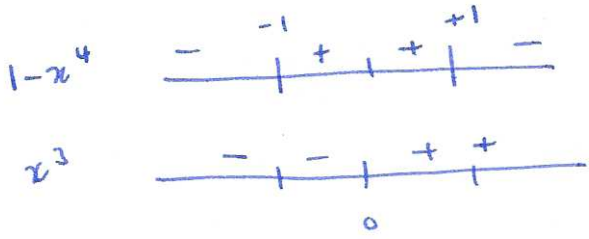
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptote : $x = 0$ $\lim_{x \to \pm\infty} f(x) = -\infty$ no horizontal asymptotes

b) $f'(x) = -3x^{-2} - 3x^2$

$f''(x) = 6x^{-3} - 6x$ solve $f''(x) = 0$

$= 6\dfrac{1 - x^4}{x^3}$ $x^{-3} = x$ $x^4 = 1$ $x = \pm 1$.



concave up $(-\infty, -1) \cup (0, 1)$

down $(-1, 0) \cup (1, \infty)$

f'' + − + −

(5) Find the area under the graph of $y = 2x^2 + 3$ between $x = -2$ and $x = 1$.

$$\int_{-2}^{1} 2x^2+3\,dx = \left[\frac{2}{3}x^3+3x\right]_{-2}^{1} = \frac{2}{3}+3-\left(\frac{2}{3}\cdot(-8)-6\right)$$

$$= \frac{2}{3}+3+\frac{16}{3}+6 = 15$$

(6) Find the indefinite integral $\int 2\cos(x) - \frac{3}{x} + \frac{4}{\sqrt{x}}\, dx.$

$2\sin(x) - 3\ln(x) + 8x^{1/2} + C$

(7) Find the indefinite integral $\int x \sin(2x^2 - 3)\, dx$.

$u = 2x^2 - 3$

$\frac{du}{dx} = 4x$

$$\int x \sin(u) \frac{dx}{du} du$$

$$\int x \sin(u) \frac{1}{4x} du$$

$$\frac{1}{4} \int \sin(u) du = -\frac{1}{4} \cos(u) + C$$

$$= -\frac{1}{4} \cos(2x^2 - 3) + C$$

(8) Find the definite integral $\int_0^1 e^x\sqrt{3e^x+1}\,dx$.

$u = 3e^x + 1$

$\frac{du}{dx} = 3e^x$

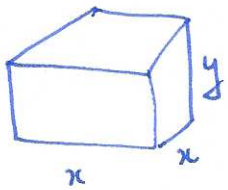
$$\int_4^{3e+1} e^x \sqrt{u}\,\frac{dx}{du}\,du$$

$$\int_4^{3e+1} e^x u^{1/2}\,\frac{1}{3e^x}\,du$$

$$\int_4^{3e+1} \frac{1}{3}u^{1/2}\,du = \left[\frac{2}{9}u^{3/2}\right]_4^{3e+1}$$

$$= \frac{2}{9}\left((3e+1)^{3/2} - 4^{3/2}\right) \approx 4.3777$$

(9) A cardboard box has a square base and no top. What dimensions minimize surface area if the total volume of the box is $2m^3$?



$V = x^2 y = 2$

$y = \frac{2}{x^2}$

$A = x^2 + 4xy$

$A = x^2 + \frac{8}{x}$

$\frac{dA}{dx} = 2x - \frac{8}{x^2}$

critical point $\frac{dA}{dx} = 0$:

$2x - \frac{8}{x^2} = 0$

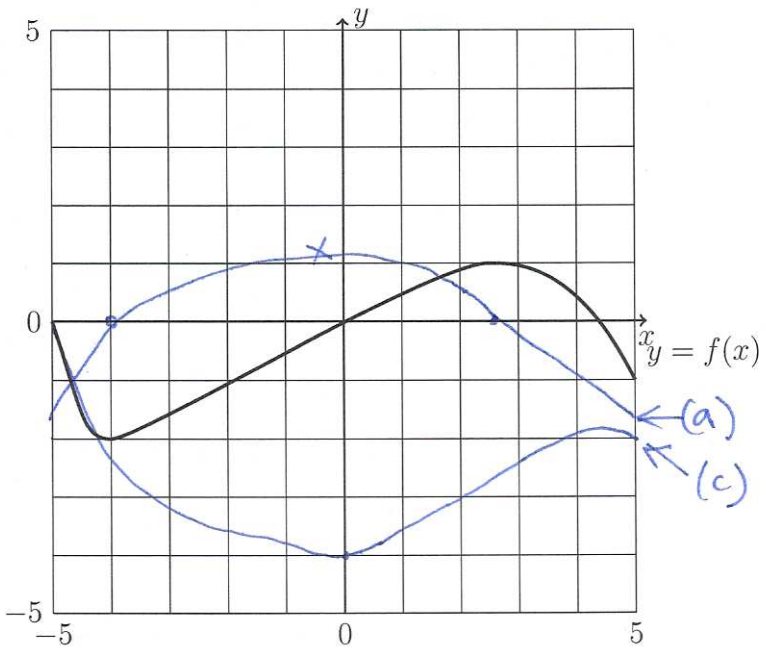
$x = \frac{4}{x^2}$

$x^3 = 4$

$x = \sqrt[3]{4}$

$y = \frac{2}{2\sqrt[3]{2}}$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. (x)

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.