# Math 231 Calculus 1 Fall 25 Midterm 2b

Name: _____Solutions_____

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a US letter page of notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 2 | |
| Overall | |

1

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^3 e^x$.

$$3x^2 e^x + x^3 e^x$$

(b) $f(x) = \dfrac{\ln(x)}{\sin(x)}$.

$$\frac{(\sin x) \cdot \frac{1}{x} - \ln(x) \cos x}{\sin^2(x)}$$

(2) (10 points) Find the derivative of the function $f(x) = \cos^{-1}(1 - 2x^2)$.

$$\frac{-1}{\sqrt{1 - (1 - 2x^2)}} \cdot (-4x)$$

(3) (10 points) Find the second derivative of the function $f(x) = \sqrt[3]{1-3x}$.

$$= \left(1-3x\right)^{1/3}$$

$$f'(x) = \frac{1}{3}\left(1-3x\right)^{-2/3}(-3)$$

$$f''(x) = -\frac{2}{9}\left(1-3x\right)^{-5/3}\cdot 9 = -2\left(1-3x\right)^{-5/3}$$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $4xy^2 + 3x^2y = 10$ at the point $(1, -2)$.

$$4y^2 + 4x \cdot 2yy' + 6xy + 3x^2y' = 0$$

at $(1, -2)$

$$16 + -16y' + -12 + 3y' = 0$$

$$4 = 13y' \qquad y' = \frac{4}{13}.$$

$$y + 2 = \frac{4}{13}(x - 1)$$

(5) Find the following limit: $\displaystyle\lim_{x\to 0}\frac{1-\cos(2x)}{1-e^{3x^2}}$

$$\overset{l'H}{=}\lim_{x\to 0}\frac{+\sin(2x)\cdot 2}{-e^{3x^2}\cdot 6x}\overset{l'H}{=}\lim_{x\to 0}\frac{\cos(2x)\cdot 4}{-e^{3x^2}\cdot(6x)^2-e^{3x^2}\cdot 6}=\frac{4}{-6}=-\frac{2}{3}$$

(6) (10 points) The area of a circular oil slick increases at the rate of 5m/min. How fast is the radius increasing when the radius is 12m?

$A = \pi r^2$

$$\underbrace{\frac{dA}{dt}}_{5} = 2\pi \underset{12}{r} \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{24\pi} \text{ m/min}$$

(7) (10 points) Use linear approximation to estimate $\sqrt{35}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(35) \approx f(36) + f'(36) \cdot (-1)$

$$6 + \frac{1}{12} \cdot -1 = 6 - \frac{1}{12}$$

percentage error:

$$\frac{\left| 6 - \frac{1}{12} - \sqrt{35} \right|}{\sqrt{35}} \times 100 \approx 0.009929\%$$

(7) (10 points) Use linear approximation to estimate $\sqrt{35}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(35) \approx f(36) + f'(36) \cdot (-1)$
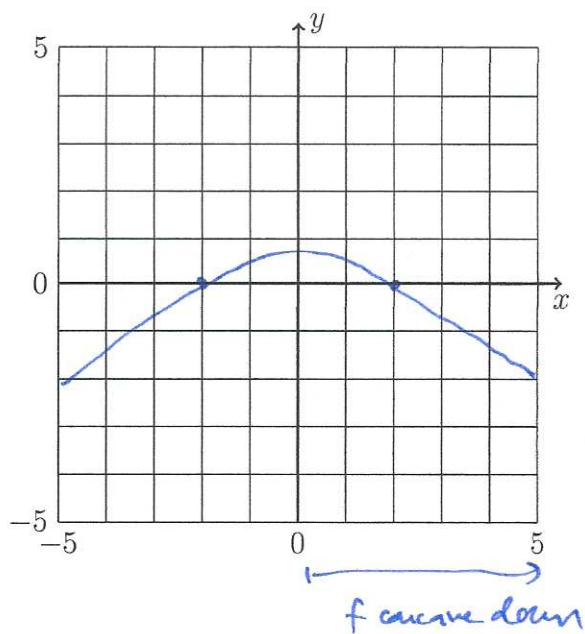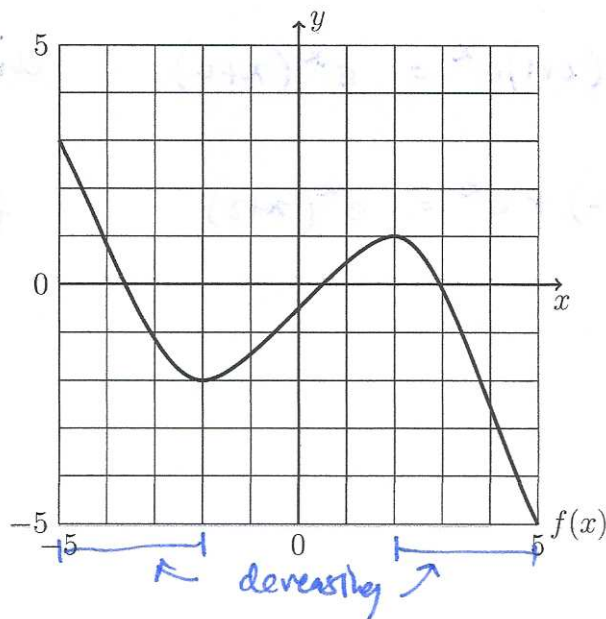
$$6 + \frac{1}{12} \cdot -1 = 6 - \frac{1}{12}$$

(8) Find the critical points for the function $f(x) = (x+1)e^x$ and use the second derivative test to classify them.

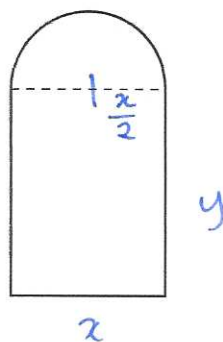$$f'(x) = e^x + (x+1)e^x = e^x(x+2)$$

critical point $f'(x) = 0$ : $x = -2$

$$f''(x) = e^x(x+2) + e^x = e^x(x+3)$$

$$f''(-2) = e^{-2} > 0$$

local min

(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$, and then use this to find where $f(x)$ is concave down.



*decreasing*



*f concave down*

(10) A window is constructed in the shape of a rectangle with a semicircle attached to the top. If the total circumference of the window is 6m, what is the largest possible area?



$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$C = x + 2y + \pi\frac{x}{2} = 6$

$y = 3 - x\left(1 + \frac{\pi}{2}\right)$

$A = x\left(3 - x\left(\frac{\pi}{2} + 1\right)\right) + \frac{\pi}{8}x^2$

$= 3x + x^2\left(-\frac{\pi}{2} - 1 + \frac{\pi}{8}\right)$ . critical point:

$\frac{dA}{dx} = 3 + 2x\left(-1 - \frac{3\pi}{8}\right)$          $\frac{dA}{dx} = 0$          $x = \dfrac{3}{2\left(1 + \frac{3\pi}{8}\right)} \approx 0.6887$

$A = \dfrac{9}{2\left(1 + \frac{3\pi}{8}\right)} + \dfrac{9}{4\left(1 + \frac{3\pi}{8}\right)} \cdot \left(-1 - \frac{3\pi}{8}\right) \approx 1.0330$