Math 231 Calculus 1 Fall 25 Midterm 2a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a US letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^2 e^x$.

$2xe^x + x^2 e^x$

(b) $f(x) = \dfrac{\cos(x)}{\ln(x)}$.

$$\frac{\ln(x).(-\sin x) - \cos x \cdot \frac{1}{x}}{(\ln(x))^2}$$

(2) (10 points) Find the derivative of the function $f(x) = \sin^{-1}(1 - 3x^2)$.

$$\frac{1}{\sqrt{1-(1-3x^2)^2}} \cdot -6x$$

(3) (10 points) Find the second derivative of the function $f(x) = \sqrt[3]{1-2x}$.

$= (1-2x)^{1/3}$

$f'(x) = \frac{1}{3}(1-2x)^{-2/3} \cdot (-2)$

$f''(x) = -\frac{2}{9}(1-2x)^{-5/3} \cdot 4$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $3xy^2 + 2x^2y = 2$ at the point $(-2, 1)$.

$3y^2 + 3x\,2yy' + 4xy + 2x^2y' = 0$

at $(-2,1)$:

$3 + -12y' - 8 + 8y' = 0$

$-4y' = 5$

$y' = -\frac{5}{4}$.

$y - 1 = -\frac{5}{4}(x+2)$

(5) Find the following limit: $\lim_{x\to 0} \dfrac{1-e^{2x^2}}{1-\cos(4x)}$

$\overset{l'H}{=} \lim_{x\to 0} \dfrac{-e^{2x^2}\cdot 2x}{\sin(4x)\cdot 4} \overset{l'H}{=} \lim_{x\to 0} \dfrac{-e^{2x^2}\cdot 4x^2 - e^{2x^2}\cdot 2}{\cos(4x)\cdot 16} = -\dfrac{2}{16} = -\dfrac{1}{8}$

(6) (10 points) The area of a circular oil slick increases at the rate of 4m/min. How fast is the radius increasing when the radius is 10m?

$A = \pi r^2$

$\underbrace{\frac{dA}{dt}}_{4} = \pi \cdot 2r \underbrace{\frac{dr}{dt}}_{\uparrow\ 10}$

$\frac{dr}{dt} = \frac{4}{2\pi} = \frac{1}{5\pi}$

(7) (10 points) Use linear approximation to estimate $\sqrt{24}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f(24) \approx f(25) + f'(25).(-1)$

$5 + \frac{1}{2.5}.-1 = 5 - \frac{1}{10} = 4.9$

percentage error: $\frac{|4.9 - \sqrt{24}|}{\sqrt{24}} \times 100 \approx 0.02\%$

(8) Find the critical points for the function $f(x) = (x-2)e^x$ and use the second derivative test to classify them.

$f'(x) = e^x + (x-2)e^x = e^x(x-1)$ critical point: $f'(x) = 0$

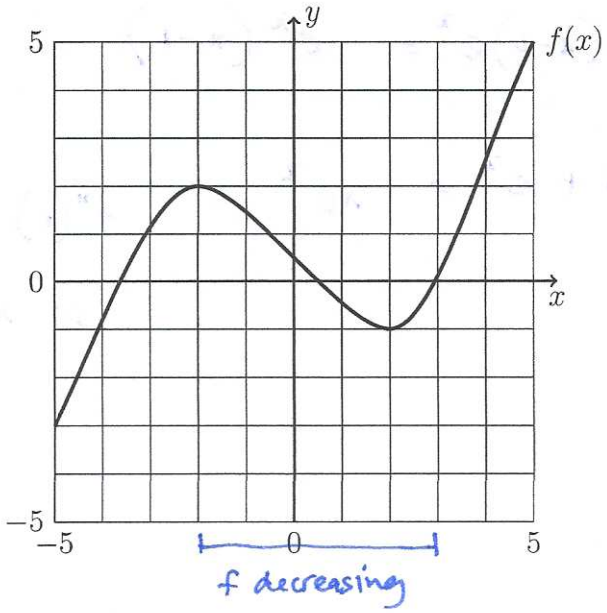
$x = 1$

$f''(x) = e^x(x-1) + e^x = xe^x$

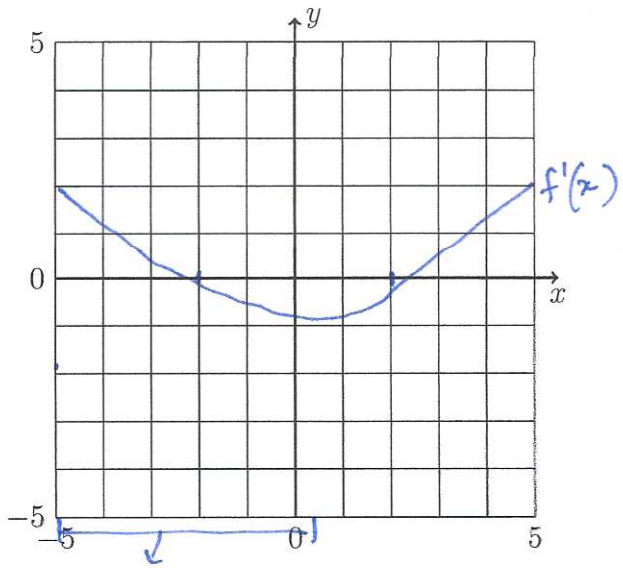
$f''(1) = e > 0$

so $x = 1$ is a local min

(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is decreasing. On the lower set of axes sketch $f'(x)$, and then use this to find where $f(x)$ is concave down.

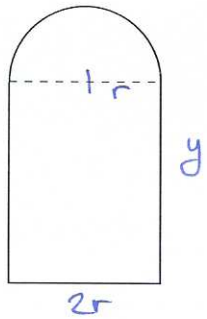


f decreasing



concave down ↔ $f''(x) < 0$ ↔ $f'(x)$ decreasing

(10) A window is constructed in the shape of a rectangle with a semicircle attached to the top. If the total circumference of the window is 4m, what is the largest possible area?



$A = 2ry + \frac{1}{2}\pi r^2$

$C = 2r + 2y + \frac{1}{2}2\pi r = 4$

$y = 2 - r\left(1 + \frac{\pi}{2}\right)$

$A = 2r\left(2 - r\left(1+\frac{\pi}{2}\right)\right) + \frac{1}{2}\pi r^2$

$A = 4r - r^2\left(2+\frac{\pi}{2}\right)$

$\frac{dA}{dr} = 4 - 2r\left(2+\frac{\pi}{2}\right)$ critical point $\frac{dA}{dr} = 0$ $r = \frac{2}{2+\frac{\pi}{2}} \approx 0.5601$

$A = 4.\left(\frac{2}{2+\frac{\pi}{2}}\right) - \left(\frac{2}{2+\frac{\pi}{2}}\right)^2\left(2+\frac{\pi}{2}\right) \approx 0.1202$