Math 231 Calculus 1 Fall 25 Midterm 1b

Name: Solutions

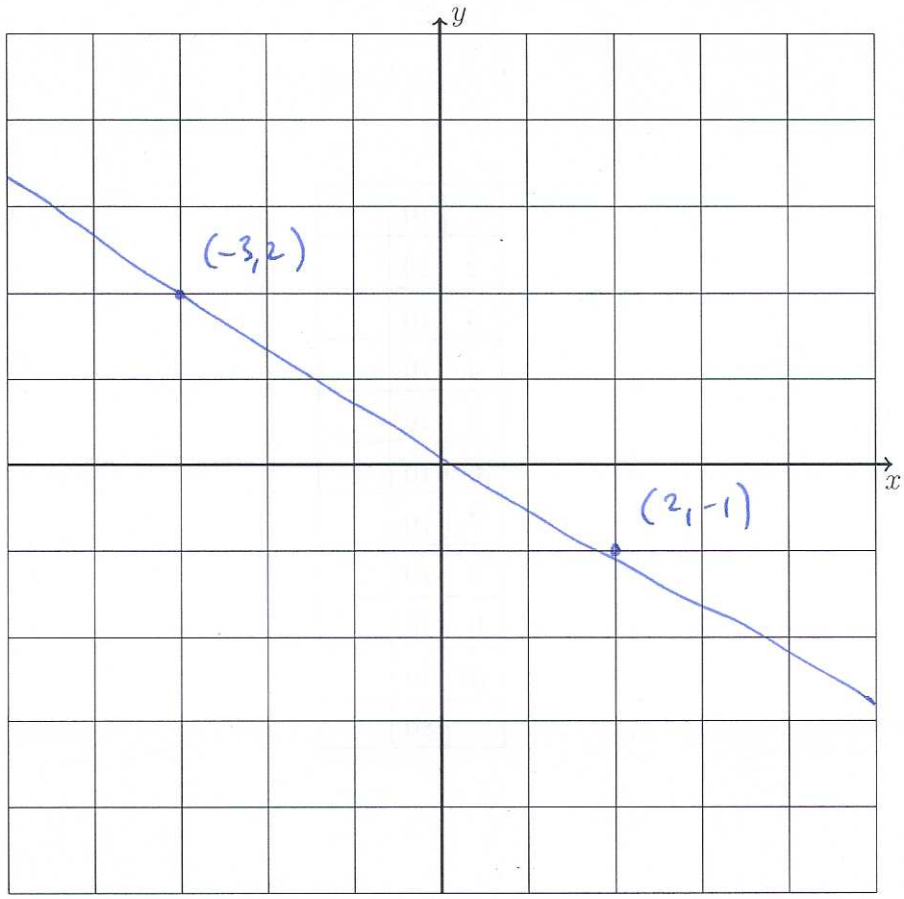
- I will count your best 8 of the following 10 questions.
- You may use a calculator, and half a US letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(-3, 2)$ and $(2, -1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

slope $m = \dfrac{-1-2}{2-(-3)} = \dfrac{-3}{5}$

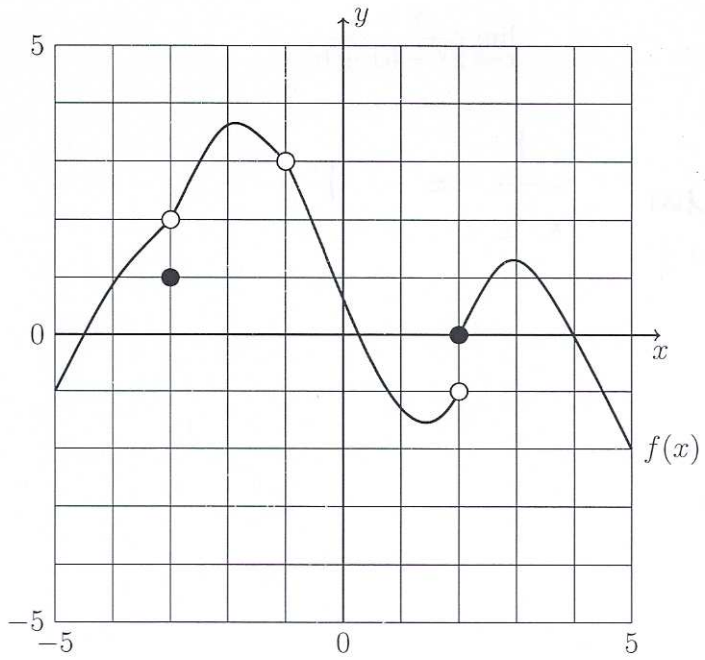


$y + 1 = -\frac{3}{5}(x-2)$

$y = -\frac{3}{5}x + \frac{6}{5} - 1$

$y = -\frac{3}{5}x + \frac{1}{5}$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x \to 2^-} f(x)$ -1

(b) $\lim_{x \to 2^+} f(x)$ 0

(c) $\lim_{x \to 2} f(x)$ DNE

(d) $\lim_{x \to -1} f(x)$ 3

(e) $\lim_{x \to -3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 3} \frac{x-3}{x^2-5x+6}$$

$$\lim_{x \to 3} \frac{(x-3)}{(x-3)(x-2)} = \lim_{x \to 3} \frac{1}{x-2} = 1$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 25} \frac{5 - \sqrt{x}}{25 - x}$$

$$\lim_{x \to 25} \frac{5-\sqrt{x}}{(5-\sqrt{2})(5+\sqrt{x})} = \lim_{x \to 25} \frac{1}{5+\sqrt{x}} = \frac{1}{10}$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x+1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x+1-(x+h+1)}{(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{1-4x^2}{\sqrt{5+4x^4}}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x^2}-4}{\sqrt{\frac{5}{x^4}+4}} = \frac{-4}{2} = -2$$

(7) Find the first and second derivatives of $f(x) = x^4 + 2\sqrt{x} - \frac{3}{\sqrt{x}}$.

$x^4 + 2x^{1/2} - 3x^{-1/2}$

$f'(x) = 4x^3 + x^{-1/2} + \frac{3}{2}x^{-3/2}$

$f''(x) = 12x^2 - \frac{1}{2}x^{-3/2} - \frac{9}{4}x^{-5/2}$

(8) Find the first and second derivatives of $f(x) = 2xe^x - \sqrt[4]{x}$.

$2xe^x - x^{1/4}$

$f'(x) = 2e^x + 2xe^x - \frac{1}{4}x^{-3/4}$

$f''(x) = 2e^x + 2e^x + 2xe^x + \frac{3}{16}x^{-7/4}$

$= 4e^x + 2xe^x + \frac{3}{16}x^{-7/4}$

(9) Find the derivative of $f(x) = \dfrac{x + e^x}{2x - 1}$.

$$f'(x) = \frac{(2x-1)(1+e^x) - (2)(x+e^x)}{(2x-1)^2}$$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.