Math 231 Calculus 1 Fall 25 Midterm 1a

Name: Solutions

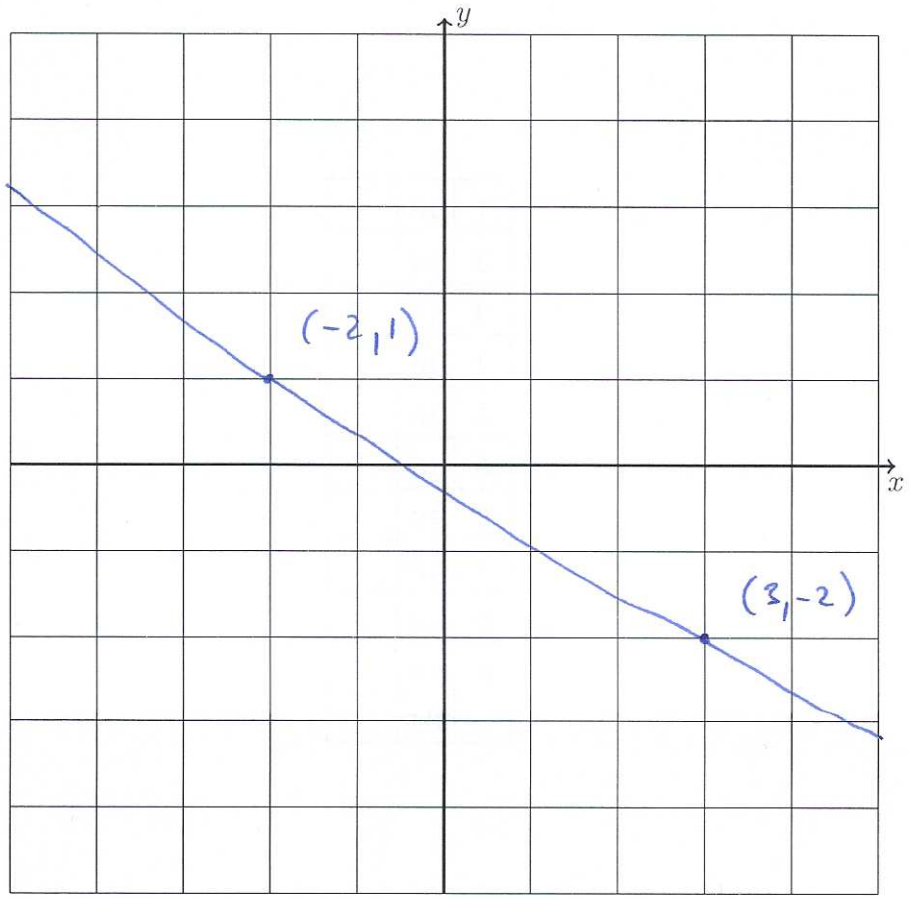
- I will count your best 8 of the following 10 questions.
- You may use a calculator, and half a US letter page of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(3, -2)$ and $(-2, 1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

slope $m = \dfrac{1-(-2)}{-2-3} = -\dfrac{3}{5}$

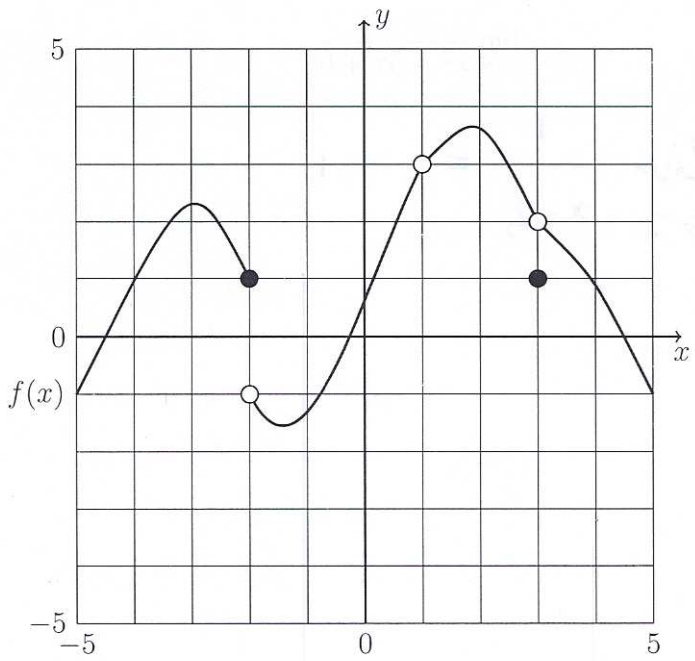


$y - 1 = -\frac{3}{5}(x+2)$

$y = -\frac{3}{5}x + 1 - \frac{6}{5}$

$y = -\frac{3}{5}x - \frac{1}{5}$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x \to -2-} f(x)$ 1

(b) $\lim_{x \to -2+} f(x)$ −1

(c) $\lim_{x \to -2} f(x)$ DNE

(d) $\lim_{x \to 1} f(x)$ 3

(e) $\lim_{x \to 3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 2} \frac{x-2}{x^2-5x+6}$$

$$\lim_{x \to 2} \frac{x-2}{(x-2)(x-3)} = \lim_{x \to 2} \frac{1}{x-3} = -1$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{2-\sqrt{x}}{x-4}$$

$$\lim_{x\to 4} \frac{(2-\sqrt{x})(2+\sqrt{x})}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{-1}{\sqrt{x}+2} = -\frac{1}{4}$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x+1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x+1-(x+h+1)}{(x+h+1)(x+1)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{4-x^2}{\sqrt{3+4x^4}}$$

$$\lim_{x\to\infty} \frac{\frac{4}{x^2}-1}{\sqrt{\frac{3}{x^4}+4}} = \lim \frac{-1}{2}$$

$x^4 - 3x^{1/2} + 3x^{-1/2}$

(7) Find the first and second derivatives of $f(x) = x^4 - 3\sqrt{x} + \frac{3}{\sqrt{x}}$.

$$f'(x) = 4x^3 - \frac{3}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$$

$$f''(x) = 12x^2 + \frac{3}{4}x^{-3/2} + \frac{9}{4}x^{-5/2}$$

$x^{1/3} - 3xe^x$

(8) Find the first and second derivatives of $f(x) = \sqrt[3]{x} - 3xe^x$.

$f'(x) = \frac{1}{3}x^{-2/3} - 3xe^x - 3e^x$

$f''(x) = -\frac{2}{9}x^{-5/3} - 3xe^x - 3e^x - 3e^x$

$= -\frac{2}{9}x^{-5/3} - 3xe^x - 6e^x$

(9) Find the derivative of $f(x) = \dfrac{3x-1}{e^x+x}$.

$$f'(x) = \frac{(e^x+x)(3) - (3x-1)(e^x+1)}{(e^x+x)^2}$$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.