Math 231 Calculus 1 Fall 25 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

    (a) $2x^3 - \frac{3}{\sqrt[4]{x^3}} + \csc(x)$

    (b) $f(x) = \dfrac{x - x^2}{\ln(3x + 2)}$

    (c) $f(x) = e^{-3x}\sin(2 - 3x)$

    (d) $f(x) = \sqrt[4]{e^{-\sin(2x)} + 3}$

(2) Evaluate the following integrals.

    (a) $\displaystyle\int \frac{3}{x^4} - 2\sin(x) + e^x \, dx$

    (b) $\displaystyle\int \frac{(2 - 3x)^2}{\sqrt{x^5}} \, dx$

    (c) $\displaystyle\int_0^{\pi/6} \sin^3(2x)\cos(2x) \, dx$

    (d) $\displaystyle\int \frac{1}{4 + x^2} \, dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE). Justify your answers.

    (a) Find $\displaystyle\lim_{x \to -3} \frac{x + 3}{x^2 + x - 6}$.

    (b) Find $\displaystyle\lim_{x \to 0} \frac{e^{2x} - 1}{\sin(3x)}$.

    (c) Find $\displaystyle\lim_{x \to 0+} x^{\sin(3x)}$.

    (d) Find $\displaystyle\lim_{x \to 0} \frac{1}{x^2} - \frac{1}{\sin^2(x)}$.
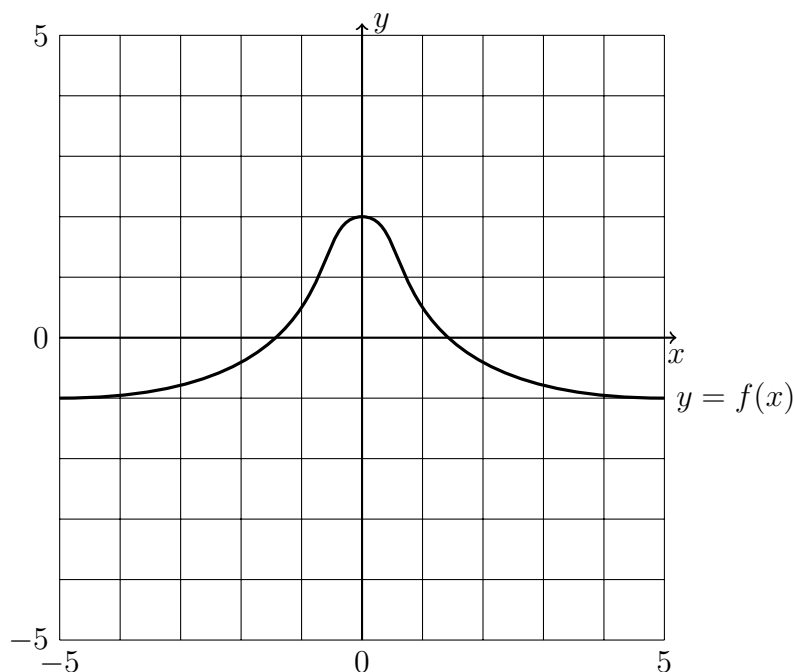
(4) Consider $f(x) = x^3 - 12x$.

    (a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.

    (b) Give the interval(s) for which f is increasing.

(c) Give the intervals for which f is concave up, and for which it is concave down.

(d) Decide which critical points are maxima, minima, or neither.

(e) Sketch the graph of $f(x)$.
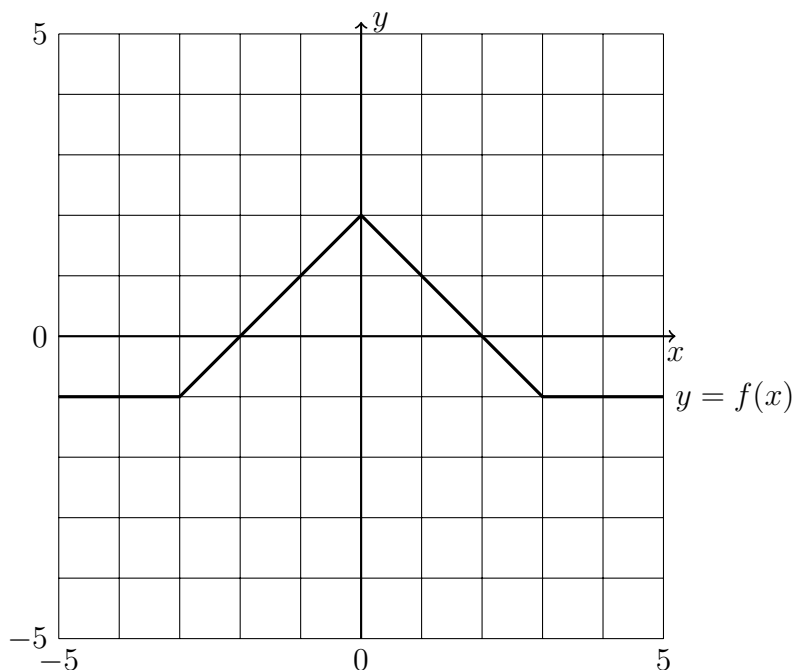
(5) Consider the function $f(x)$ defined by the following graph.

(a) Label all regions where $f(x) < 0$.

(b) Label all regions where $f'(x) > 0$.

(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{3}{3 - x}$.

(a) Sketch the graph of $f(x)$ showing any asymptotes.

(b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.

(c) Sketch the tangent line at $x = -1$ on your graph.

(7) Let $f(x) = \frac{1}{x} - x$. Find the derivative *using the limit definition of the derivative*. Do not use L'Hôpital's rule. Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y^2 + 3x - 2y = 5$ at the point $(-2, -1)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.

(10) A region in the plane is bounded by the x-axis, the graph $y = 9 - x^2$, and the lines $x = -1$ and $x = 1$.
  (a) Sketch the region (shading it in) and label the boundaries.
  (b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $2\text{in}^3/\text{s}$. How fast is the volume growing when $r = 6\text{in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

(12) Use linear approximation to estimate $\sqrt[3]{26}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) What's the closest point on the line $y = 2x + 4$ to the point $(2, -1)$?