

MTH 301: Introduction to Proof, Fall 2024

M 12:20 - 2:15 1S-102
W 12:20 - 2:15 1S-102

Section 29492

Instructor: Joseph Maher
Office: 1S-222
Office hours: M 2:30-4:35, W 2:30-3:20
Webpage: <http://www.math.csi.cuny.edu/~maher/teaching>
Email: joseph.maher@csi.cuny.edu
Phone: (718) 982-3623

Important Dates: <http://www.csi.cuny.edu/currentstudents/academiccalendars/>

Text: Fendel and Resek, *Foundations of Higher Mathematics*, Pearson ISBN 9780201125870.

Help: Email is the best way to contact me. Come and see me during my office hours! Free math tutoring is also available <http://www.math.csi.cuny.edu/Students/Tutoring>.

Disability policy: Qualified students with disabilities will be provided reasonable academic accommodations if determined eligible by the Center for Student Accessibility. Prior to granting disability accommodations in this course, the instructor must receive written verification of student's eligibility from the Center for Student Accessibility, which is located in 1P-101. It is the student's responsibility to initiate contact with the Center for Student Accessibility staff and to follow the established procedures for having the accommodation notice sent to the instructor.

Integrity policy: CUNY's Academic Integrity Policy is available online at <https://www.csi.cuny.edu/catalog/undergraduate/academic-policies.htm#o3518>

No late homework will be accepted.

Aim of the course

The goals of this course are for you to become adept at logical reasoning and to increase your deep thinking stamina by learning some central mathematics and mathematical processes that are essential for studying higher mathematics. We will focus especially on the idea of a mathematical proof. You will have succeeded in this course if by the end of the semester you understand or have learned

- properties of sets, integers, functions, proof by induction, relations and partitions covered in class
- existential and universal quantifiers; translating between English and symbolic sentences using quantifiers
- if-then statements: when they are true, what they are saying / not saying, what constitutes a counterexample
- how the appearance of quantifiers in the conclusion of an if-then statement guides the structure of its proof
- how to think deeply, carefully, and logically about new concepts
- how to explore a concept, formulate your own questions about the concept as a result of your exploration, and use proof techniques to prove or disprove these questions with certainty

- how to express your ideas clearly, both in English and using the language of mathematics

Supplies

Text: Foundations of Higher Mathematics, by Fendel and Resek.

Stapler: All assignments more than one page long must be stapled, or they will not be accepted.

Grading

Attendance/Participation	10%
Writing assignments	5%
Homework	20%
Midterm 1	20%
Midterm 2	20%
Final	25%

This course may be different from other math courses you have taken. Mathematics and mathematical reasoning are best learned by doing; for that reason, you will spend a considerable amount of time during class working in small groups with other students, investigating and discussing mathematical problems - this is not mainly a lecture course! Your homework time will be spent doing mathematical problems and investigations for which a solution can not be obtained by following an example from a book, but instead will require a lot of time, investigation, trial and error, concentrated thought, and eventually deep understanding. In addition, you will be asked to write up your findings very clearly and precisely, using full sentences.

Frequently asked questions about the syllabus

Why do you care about attendance and participation?

In this class you will be investigating mathematics with a small group of your classmates, and the group will depend on you for input of ideas and moral support. This means you need to regularly attend class and arrive on time, prepared and willing to contribute. Participating in class and in your group means that you discuss the problems with your groupmates, asking questions when you are confused, and explaining your ideas (kindly and respectfully) about the problems, whether you are certain they are correct or not.

What does it mean to be prepared for class?

In order to succeed in this class, you absolutely must prepare yourself for each class period by reading (see below) the assigned section of the text before class, bringing your text to class, being willing to think deeply on your own about each problem, and being awake enough during class that you can communicate your ideas to your groupmates.

Why are there writing assignments in a math class?

They will give you much needed practice, especially in a math class, in writing and expressing your thoughts using well-written sentences. Secondly, the questions I will ask you to write about are intended to prompt you to think more deeply about mathematics and the way you learn mathematics, and they will also give me more personal insight into how the class is going for you. And finally, it offers you a forum to express your concerns, comments, and suggestions.

Do we really have to do homework?

Yes, and you will find it extremely helpful, rewarding, and even fun. One of the goals of this class is to learn how to communicate effectively about mathematics; this can only be acquired through

practice and effort. Moreover, you will not fully understand a mathematical concept until you have applied your knowledge to a variety of interesting problems. For this reason, written homework will be assigned almost every class period and will be due the next class period, and reading assignments will be given each day. It is imperative for your success in this class that you read and think about the material in the book on time. I cannot stress this enough.

Homework must be neatly written, using full clear sentences, good grammar, and a staple. Please leave adequate blank space on your homework that the grader has room to make constructive comments. Late homework will not be accepted under any circumstances; if you must miss class, you may turn your homework assignment in early, send it in with a friend, or write it in pen, scan it in and email it to me.

I encourage you to discuss strategies for solving the homework problems with other students; however, your final write-up of the problems must be done entirely on your own. Turning in a solution that was even partially copied from another student, or a solution which was partially developed line-by-line in conjunction with another student or students, constitutes an Academic Integrity violation, and will be treated as such.

Will this be on the test?

Yes.

When I see new questions I feel lost and don't know how to get started.

This is normal. We will explore lots of strategies for starting to work on problems when you don't know in advance exactly what method to use, or what the solution is going to look like.

I get frustrated when I can't do the problems.

This is normal. When you go to the gym to take exercise your body gets hot and sweaty. In the same way, when you work at discovering new mathematics, your brain also has to work hard and you get tired and frustrated. When you get over-tired, then you should take a nap or sleep, and continue when you are refreshed.

What should I put in my class notes?

- I will spend some portion of the class giving notes, which you should carefully write down. In addition, during lecture you should also write down any ideas you have about the material, or questions you want to ask (if you didn't ask them in class), or topics you want to explore more. Put stars in your notes beside things you feel a little confused about.
- The rest of the time in class you will spend doing classwork with your groupmates. Many students treat their notebook simply as scratch paper at this point, but this is a mistake. You should write down a lot of coherent, careful notes during this as well. Write down ideas that don't work out, things you've tried, sample proofs, notes to yourself about what went wrong with your sample proofs, work out examples. Use complete sentences or at least use words and explain to yourself what you are thinking! This has two benefits: first, it will give you essential practice in turning nebulous ideas in your head into coherently written ideas (the single most challenging aspect of this class), and second, the more carefully you organize your own thoughts in this portion of your notebook, the more you will get out of it when you are studying for the exams.
- In particular, if you are trying to write a proof in your notes, write it as if you were turning it in for a grade – do not take shortcuts! You must practice good proof writing in order to be able to write good proofs on exams.

How do I read a math book?

You may have tried reading math books in the past and become frustrated. The problem is, you were probably trying to read it as you would read a novel, and that just doesn't work with mathematics. Mathematics is written incredibly precisely, carefully, and efficiently, and a casual reading will not allow you to get the full understanding of the concepts being presented. So instead:

- Any time you encounter a new concept, definition, or theorem in your reading, you must play with it to get familiar with it. This means that you stop reading, get out your pencil, and try to find things that are examples of the concept or theorem, try to find things that aren't examples and understand what goes wrong - that sort of thing. That is the only way to get real understanding of the concept. If you just read a definition without this sort of exploring (see below) and then try to read the rest of the book, you will be confused and frustrated. Passive reading is not enough!
- In addition to the above advice, which applies to any math book, in this class reading a section also means you must work on each Exploration and GYHD in the section for at least 5 minutes each.

How can someone “explore” mathematics?

Mental exploration means examining a situation, with or without particular question in mind, and discovering whatever you can about it. As described above, you will have to get out your pencil, think up examples and see how they behave, try to see relationships, and write out your observations in words.

- Be aware that even when you are given a concrete question to explore, you will not get answers right away. You will probably not even understand the question right away!
- Your focus should be on the process of thinking and understanding rather than on the final answer. Resist urge to look ahead to see if you can find answer in this book or another. You will have squandered all your opportunity to learn from the exercise if you do this.
- Bouncing ideas off of other people helps you gain perspective and insight, even (especially) if that other person is not an expert in the topic. So, it will be helpful to you if you can find someone else (classmate, roommate, friend, mother?) to talk about your ideas with.
- Exploration is assisted by good record keeping. Write down in words what you were thinking when you chose your example, what you discovered, and how that lead you to your next example.