

Math 301 Introduction to Proof Fall 24 Sample Midterm 1

- (1) Find three distinct elements for the truth sets of the following statements:
- (a) $xy = 1$, where the universe is $\mathbb{R} \times \mathbb{R}$.
 - (b) A is a subset of \mathbb{N} which is closed under multiplication.
 - (c) A is an element of $\mathcal{P}(\mathbb{N})$.
 - (d) A is a subset of $\mathcal{P}(\mathbb{N})$.
 - (e) The numbers a, b and c are the lengths of the sides of a right angled triangle, where the universe is $\mathbb{R}_{>0} \times \mathbb{R}_{>0} \times \mathbb{R}_{>0}$.
- (2) Consider the statement: If $a|b^2$ then $a|b$.
Which, if any, of the following substitutions give a counterexample.
a) $a = 2, b = 4$ b) $a = 3, b = 4$ c) $a = 8, b = 4$.
- (3) Write out a careful proof of the fact that the sum of a number with its square is even.
- (4) What is $\mathcal{P}(\{\emptyset\})$? What is $\mathcal{P}(\mathcal{P}(\{\emptyset\}))$?
- (5) If $A \cup B \subseteq A \cup C$ does this imply that $B \subseteq C$?
- (6) State which of the following statements, are true, vacuously true, or false.
- (a) If $A \subseteq A \cap B$, then $A \subseteq B$.
 - (b) If $\mathcal{P}(A) = \emptyset$, then $A = \emptyset$.
 - (c) If $A \notin B$ and $B \notin C$, then $A \notin C$.
 - (d) $(A \setminus B') \cap C = A \cap (B \cup C)$
- (7) Suppose A and B are finite sets with $|A| = a$, $|B| = b$ and $|A \cap B| = c$. Find
- (a) $|B \setminus A|$
 - (b) $|A \times (A \cup B)|$
 - (c) $|\mathcal{P}(A \setminus B)|$
- (8) There is an island where all people either always lie, or always tell the truth. You meet three people, A , B and C .
 A says: At least one of us is a truth teller.
 B says: A is lying.
 C says: B is lying.
What can you deduce?
- (9) There is an island where all people either always lie, or always tell the truth. You meet D who says "I am lying." What can you deduce?