MTH301 Fall 24 SMT1 Solutions

Q1 a) $(1,1)$, $(-1,-1)$, $(2,\frac{1}{2})$.

b) $\mathbb{N}$, $\{1\}$, $\{2,4,\cancel{6},\overset{8}{\cancel{16}},\dots\} \simeq 2\mathbb{Z}$.

c) ϕ, $\{1\}$, $\mathbb{N}$

d) $\{\phi\}$, $\{\{1\}\}$, $\{\mathbb{N}\}$.

e) $(3,4,5)$ $(5,12,13)$ $(1,1,\sqrt{2})$.

Q2 c)

Q3 Thm If $x \in \mathbb{Z}$ then x^2+x is even.

Proof We consider two cases depending on whether x is even or odd. Case 1: If x is even then $x = 2n$ for some $n \in \mathbb{Z}$. Then $x^2+x = (2n)^2+2n = 4n^2+2n = 2(2n^2+n)$ which is even. Case 2: If x is odd then $x = 2n+1$ for some $n \in \mathbb{Z}$. Then $x^2+x = (2n+1)^2+(2n+1) = 4n^2+4n+1+2n+1 = 4n^2+6n+2 = 2(2n^2+3n+1)$, which is even. So in either case x^2+x is even, as required. $\square$.

Q4 $P(\{\phi\}) = \{\phi, \{\phi\}\}$. $P(P(\{\phi\})) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$.

Q5 False: $A = B = \{1\}$, $C = \phi$ counterexample.

Q6 a) T b) VT c) F d) F.

Q7 a) $b-c$ b) $a(a+b-c)$ c) 2^{a-b}

Q8

A	B	C	≥1 T.	A is lying	B is lying	
T	T	T	T	F #		
T	T	F	T	F #		
T	F	T	T	F	T	← only possibility
T	F	F	T	F	T #	
F	T	T	T #			
F	T	F	T #			
F	F	T	T #			
F	F	F	F ·	T #		

↑ if A is F can't make T statement

Q9: contradictory statement.