Function Practice Problems Fun Pack

1. Suppose $f : X \to Y$. Prove that
   (a) If $A \subseteq B \subseteq X$, then $f(A) \subseteq f(B)$
   (b) If $C \subseteq D \subseteq Y$, then $f^{-1}(C) \subseteq f^{-1}(D)$.
   (c) For every $x \in X$, $f(\{x\}) = \{f(x)\}$.

2. Suppose $f : X \to Y$, and $A, B \subseteq X$. Prove that
   (a) $f(A \cup B) = f(A) \cup f(B)$
   (b) $f(A \cap B) \subseteq f(A) \cap f(B)$
   (c) Prove or disprove: $f(A) \cap f(B) \subseteq f(A \cap B)$

3. Suppose $f : X \to Y$, and $A, B \subseteq Y$. Prove that
   (a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
   (b) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
   (c) $f^{-1}(A - B) = f^{-1}(A) - f^{-1}(B)$.

4. Suppose $f : X \to Y$ and $B \subseteq Y$. Prove that
   (a) $f[f^{-1}(B)] \subseteq B$.
   (b) Prove or disprove: $B \subseteq f[f^{-1}(B)]$
   (c) If f is onto, then $f[f^{-1}(B)] = B$

5. Suppose $f : X \to Y$. Prove that
   (a) For all $A \subseteq X$, $A \subseteq f^{-1}[f(A)]$.
   (b) Find a counterexample to the statement: For all $A \subseteq X$, $A = f^{-1}[f(A)]$
   (c) If f is 1-1, then for all $A \subseteq X$, $A = f^{-1}[f(A)]$
   (d) Suppose for all $A \subseteq X$, $A = f^{-1}[f(A)]$. Prove that f is 1-1.

6. Suppose $f : X \to Y$. Prove that
   (a) If f is 1-1, then for all $A, B \subseteq X$, $f(A \cap B) = f(A) \cap f(B)$.
   (b) Suppose that for all $A, B \subseteq X$, $f(A \cap B) = f(A) \cap f(B)$. Prove that f is 1-1.