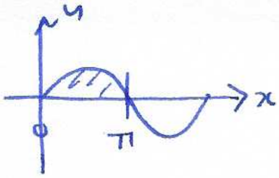
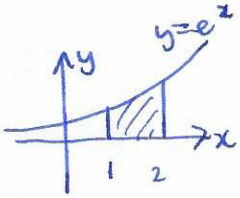
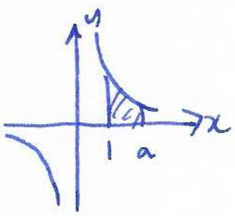


$$\int_0^{\pi} \sin(x)\,dx = \Big[-\cos x\Big]_0^{\pi} = -\cos(\pi)+\cos(0) = -(-1)+1 = 2$$

$$\int_0^{2\pi} \sin(x)\,dx = \Big[-\cos x\Big]_0^{2\pi} = -\cos(2\pi)+\cos(0) = -1+1 = 0$$



$$\int_1^2 e^x\,dx = \big[e^x\big]_1^2 = e^2 - e$$



$$\int_1^a \frac{1}{x}\,dx = \big[\ln|x|\big]_1^a = \ln(a) - \ln(1) = \ln(a)$$

<u>Observations</u>

① choice of antiderivative doesn't matter, let $F(x)$ and $F(x)+c$ be antiderivatives for $f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

$$= (F(b)+c) - (F(a)+c) = F(b) - F(a)$$

② $\int_a^t f(x)\,dx$ is a function of t! (and a, f) but <u>not</u> x. (called a dummy variable) i.e. $\int_a^t f(x)\,dx = \int_a^t f(y)\,dy$. If you want a function of x write $\int_a^x f(t)\,dt$

<u>§5.4 Fundamental theorem of calculus II</u>

<u>Thm</u> (FTC ②) Let $f(x)$ be a continuous function on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is an antiderivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx}$, so $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. Furthermore $A(a) = 0$ $\square$.

<u>Example</u> $\frac{d}{dx}\int_0^{x^2} \cos(t)\,dt$ ← use chain rule!

<u>§5.6 Substitution / change of variable</u>

"reverse chain rule for integration".