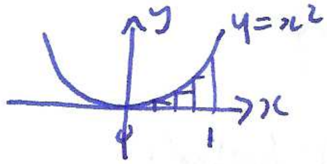
midpoint rectangles $M_N = \sum_{i=1}^{N} f(a+(i-\frac{1}{2})\Delta x)\Delta x$

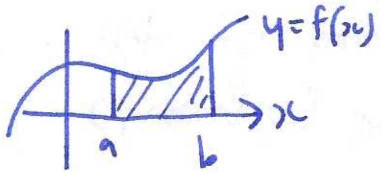
Q: what about $y=x^2$?



need to find $1^2+2^2+\dots+n^2 = \frac{1}{6}n(n+1)(2n+1)$

need better way...

§5.2 Definite integral



want: area under the curve $y=f(x)$ between $x=a$ and $x=b$

notation: $\int_a^b f(x)\,dx$

Formal def^{n}: Riemann sum $R(f, P, c)$

P: partition

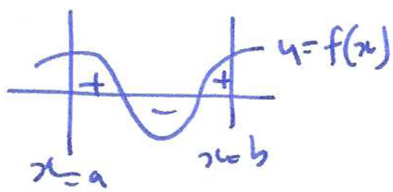
$a = x_0 < x_1 < x_2 < \dots < x_n = b$

$\Delta x_i = x_i - x_{i-1}$

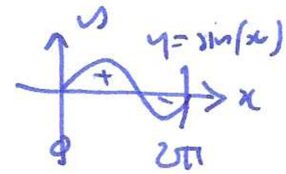
$c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\|\Delta x_i\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

note: signed area!

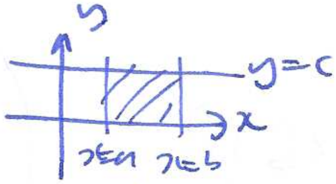


so $\int_0^{2\pi} \sin(x)\,dx = 0$



useful properties

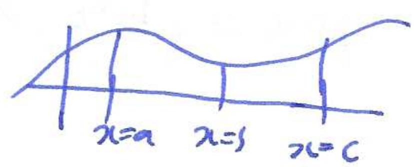
$\int_a^b c\,dx = c(b-a)$



sums: $\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

constant multiple: $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

adjacent intervals: $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$



0-length interval: $\int_a^a f(x)\,dx = 0$

reversing limits: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$