Example sketch graph of $f(x) = \dfrac{3x+2}{2x-4}$

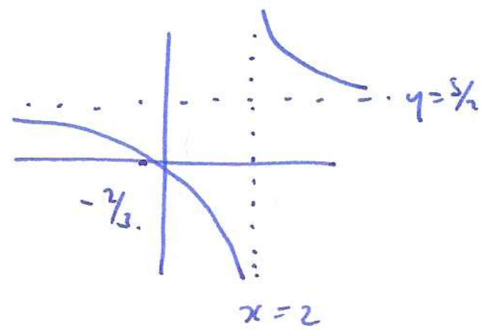
① find vertical asymptotes $\leftrightarrow$ denominator zero : $2x-4=0$ $\quad x=2$

② find $f'(x) = \dfrac{(2x-4).3-(3x+2).2}{(2x-4)^2} = \dfrac{-16}{(2x-4)^2} = \dfrac{-4}{(x-2)^2}$ $\leftarrow$ always $-$ve.

f decreasing, no critical points, except vertical asymptote at $x=2$.

③ $f''(x) = \dfrac{8}{(x-2)^3}$ $\quad$ +ve $\quad x>2$ $\quad$ concave up

$\quad$ −ve $\quad x<2$ $\quad$ concave down



④ horizontal asymptotes: $\dfrac{3x+2}{2x-4} = \dfrac{3+2/x}{2-4/x} \to \dfrac{3}{2}$

behaviour near asymptote/zeros:

		$-2/3$		2	
$3x+2$	$-$		$+$		$+$
$2x-4$	$-$		$-$		$+$
$f(x)$	$+$		$-$		$+$

Example sketch graph of $f(x) = \dfrac{x}{\sqrt{x^2+1}} = x(x^2+1)^{-1/2}$

① vertical asymptotes: none.

② find $f'(x) = \dfrac{\sqrt{x^2+1}\,.\,1 - x.\frac{1}{2}(x^2+1)^{-1/2}.2x}{x^2+1} = \dfrac{x^2+1-x^2}{(x^2+1)^{3/2}} = (x^2+1)^{-3/2} > 0.$

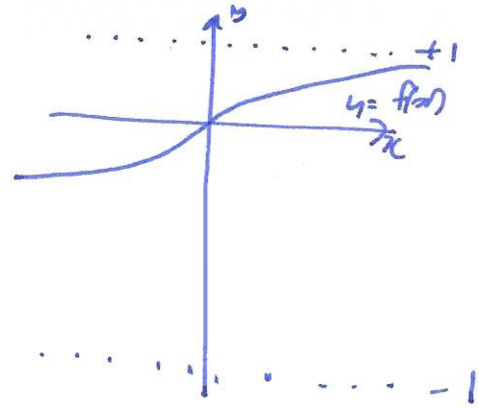
$\Rightarrow$ f increasing, no critical points.

③ $f''(x) = -\frac{3}{2}(x^2+1)^{-5/2}2x$ $\quad$ inflection point at $x=0$ $\quad$ +ve for $x<0$

$\quad$ −ve for $x>0$

④ horizontal asymptotes $\displaystyle\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}} \overset{l'H}{=} \lim_{x\to+\infty} \frac{1}{\frac{1}{2}(x^2+1)^{-1/2}.2x} = \lim_{x\to\infty} \sqrt{1+1/x^2} = 1$

$$\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x\to +\infty} \frac{-x}{\sqrt{x^2+1}} = -1$$

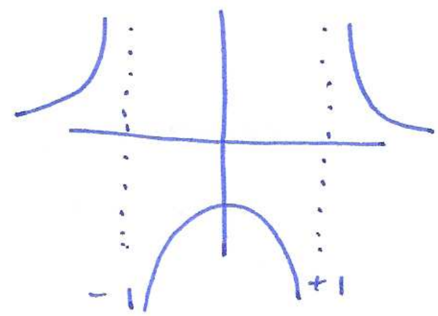


<u>Example</u> $f(x) = \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$

① vertical asymptotes at $x = \pm 1$

② $f'(x) = -(x^2-1)^{-2} \cdot 2x$ critical point at $x=0$

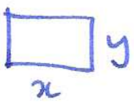
③ $f''(x) = \frac{6x^2+2}{(x^2-1)^3}$



④ etc..

§4.7 Optimization

<u>Example</u> a piece of wire of length L is bent into a rectangle. What is the largest possible area?



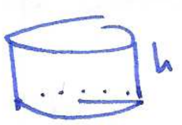
area $= xy$
length $L = 2x+2y$ $\Big\}$ $y = \frac{L}{2} - x$

$$A = x\left(\frac{L}{2} - x\right) = \frac{L}{2}x - x^2$$

$A'(x) = \frac{L}{2} - 2x$ critical point $x = \frac{L}{4}$ (local max)

so max occurs when $x = y = \frac{L}{4}$ and area is $\frac{L^2}{16}$.

<u>Example</u> what shape of cylindrical can minimises surface area, if you want total volume to be $1\,ft^3$?



$V = \pi r^2 h = 1 \Rightarrow h = \frac{1}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r \cdot \frac{1}{\pi r^2} = 2\pi r^2 + \frac{2}{r}$