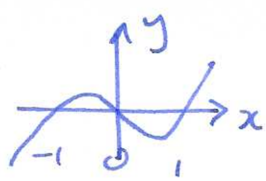
<u>Example</u> $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$

$f'(x) = 3x^2 - 1$

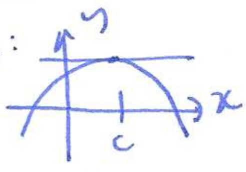
$f''(x) = 6x$



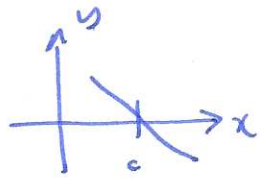
$\left.\begin{array}{l} f''(x) > 0 \text{ for } x > 0 \\ f''(x) < 0 \text{ for } x < 0 \end{array}\right\}$ $x=0$ is inflection point.

<u>second derivative</u>

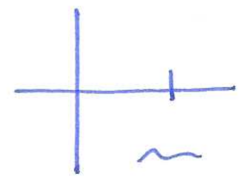
local max:



$f(x)$

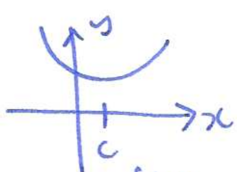


$f'(x)$

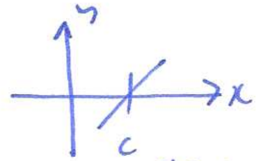


$f''(x) < 0$

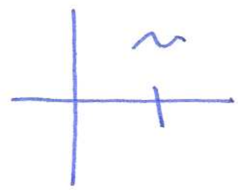
local min:



$f(x)$



$f'(x)$



$f''(x) > 0$

<u>Thm</u> Suppose $f(x)$ is differentiable and c is a critical point

if $f''(x) > 0 \Rightarrow c$ is local min

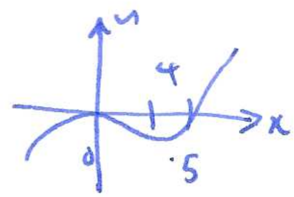
$f''(x) < 0 \Rightarrow c$ is local max

$f''(x) = 0$ NO INFORMATION (may be local max / min / neither)

<u>Example</u> $f(x) = x^5 - 5x^4 = x^4(x-5)$

$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$

$f''(x) = 20x^3 - 60x^2$



critical points: $f'(x) = 0 \Rightarrow x = 0, 4$

2nd derivative test: $x=0$ $f''(0) = 0$ no information

$x=4$ $f''(4) = 320 > 0$ local min

at $x=0$ use first derivative test:

	0	
	–	–
$(x-4)$		
x^3	–	+
$f'(x)$	+	–

$\Rightarrow$ local max.

§4.5 L'Hôpital's rule

Thm Suppose $f(x)$ and $g(x)$ are differentiable, and $f(a) = g(a) = 0$
or $f(a) = g(a) = \pm\infty$

then $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$, provided this limit exists.

warning ① this is not the quotient rule!

② $\frac{f(a)}{g(a)}$ must be indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ not $\frac{1}{0}$, i.e. can't use l'Hôpital on $\lim\limits_{x\to 0} \frac{1}{x}$.

Examples ① $\lim\limits_{x\to 1} \frac{x^3-1}{x-1} = \lim\limits_{x\to 1} \frac{3x^2}{1} = 3$

② $\lim\limits_{x\to 1} \frac{x^{100}-1}{x-1} = \lim\limits_{x\to 1} \frac{100x^{99}}{1} = 100$

③ $\lim\limits_{x\to \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1} = \lim\limits_{x\to \frac{\pi}{2}} \frac{-2\cos x \sin x}{\cos x} = \lim\limits_{x\to \frac{\pi}{2}} -2\sin x = -2$

④ $\lim\limits_{x\to 0+} x\ln(x) = \lim\limits_{x\to 0+} \frac{\ln(x)}{1/x} = \lim\limits_{x\to 0+} \frac{1/x}{-1/x^2} = \lim\limits_{x\to 0+} -x = 0$

⑤ $\lim\limits_{x\to 0} \frac{e^x - x - 1}{\cos x - 1} = \lim\limits_{x\to 0} \frac{e^x - 1}{-\sin x} = \lim\limits_{x\to 0} \frac{e^x}{-\cos x} = -1$

⑥ $\lim\limits_{x\to 0} \frac{1}{\sin x} - \frac{1}{x} = \lim\limits_{x\to 0} \frac{x - \sin x}{x\sin x} = \lim\limits_{x\to 0} \frac{1-\cos x}{\sin x + x\cos x} = \lim\limits_{x\to 0} \frac{\sin x}{\cos x + \cos x - x\sin x} = 0$

⑦ $\lim\limits_{x\to 0+} x^x = \lim\limits_{x\to 0+} e^{x\ln x}$ note: e^x cts so $= e^{\lim_{x\to 0} x\ln x} = e^0 = 1$.

Comparing growth rate of functions

Q: which grows faster $(\ln(x))^2$ or $\sqrt{x}$?

$\lim\limits_{x\to\infty} \frac{\sqrt{x}}{(\ln(x))^2} = \lim\limits_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2\ln(x)\frac{1}{x}} = \lim\limits_{x\to\infty} \frac{\sqrt{x}}{4\ln(x)} = \lim\limits_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{4/x} = \lim\limits_{x\to\infty} \frac{\sqrt{x}}{8} = \infty$

so $\sqrt{x}$ grows faster.