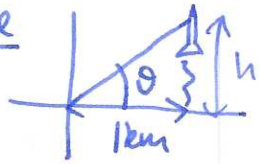
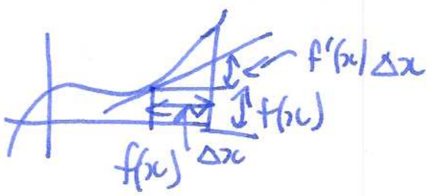
advice: ① give things names
② write down relations between them and use implicit differentiation
③ plug in numbers as necessary

Example 

if angle is $\theta = \frac{\pi}{3}$ and rate of change is $\frac{d\theta}{dt} = \frac{1}{2}$ radian/sec
how fast is the rocket going?

$\frac{h}{1} = \tan\theta \quad \frac{dh}{dt} = \sec^2\theta \frac{d\theta}{dt} \qquad \frac{dh}{dt} = \sec^2\left(\frac{\pi}{3}\right)\frac{1}{2} \approx 0.1$ km/sec

§4.1 Linear approximation



If $f(x)$ is differentiable at x, and Δx is small,
then $\quad f(x+\Delta x) \approx f(x) + f'(x)\Delta x$
so change in f is $\Delta f \approx f(x+\Delta x) - f(x)$
$\approx f'(x)\Delta x$

Example estimate $\sqrt{103}$

$f(x) = \sqrt{x} = x^{1/2} \qquad f(100) = 10 \qquad$ so $\Delta f \approx f'(x)\Delta x$

$f'(x) = \frac{1}{2}x^{-1/2} \qquad f'(100) = \frac{1}{20} \qquad \frac{1}{20}\cdot 3$

so $\sqrt{103} \approx 10 + \frac{3}{20} = 10.15$

Example you make an 18" pizza. If the diameter is accurate to ±0.4in
how much pizza do you gain/lose?

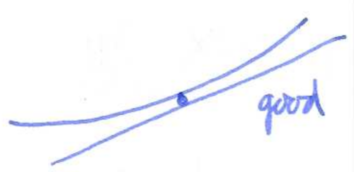
$A = \pi r^2, \quad 2r = D, \quad A = \pi\left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}, \quad A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2}$

$\Delta A = A'(18)\cdot\Delta D = \frac{1}{2}\pi \cdot 18 \cdot 0.4 \approx 11 \text{in}^2.$

Q: is this good or bad? absolute error = 11.

percentage error = $\left|\frac{\text{absolute error}}{\text{actual value}}\right| \times 100 = \frac{11}{\pi \cdot 18^2/4} \times 100 \sim 4\%.$

Observation: when is the linear approximation a good approximation?

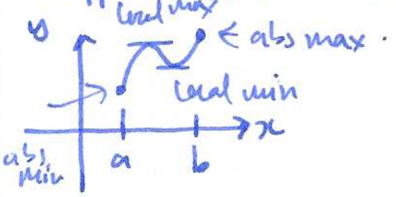


$f''(x)$ small

$f''(x)$ big.

§4.2 Extreme values

suppose $f(x)$ is defined on a closed interval $[a,b]$



Def^n $f(c)$ is the <u>absolute max</u> if $f(c) \geq f(x)$ for all $x \in [a,b]$

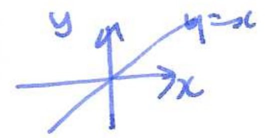
$f(c)$ is the <u>absolute min</u> if $f(c) \leq f(x)$ for all $x \in [a,b]$

note: Q: <u>where</u> is the local/abs max/min ← want x-value

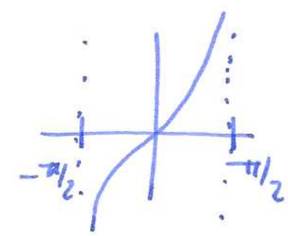
Q: <u>what</u> is the local/abs max/min ← want y-value

warning some functions do not have any local max or min

Examples $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x$



$f: (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$, $x \mapsto \tan(x)$



Thm If $f(x)$ is cts on a <u>closed and bounded</u> interval then $f(x)$ has both an absolute max and an absolute min.

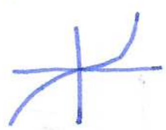
Def^n $f(x)$ has a <u>local max</u> at $x=c$ if there is ~~any~~ small interval containing c s.t. $f(x)$ is an abs max on this interval.

$f(x)$ has a <u>local min</u> at $x=c$ if there is a small interval containing c s.t. $f(x)$ is an abs min on this interval.

Def^n we say that $x=c$ is a critical point if $f'(c)=0$ (or undefined)

warning $f'(c)=0 \not\Rightarrow c$ is a local max or min.

Example $y=x^3$



$f'(x) = 3x^2$ $f'(0)=0$ but $x=0$ not local max or min

- How to find the absolute max or min of a differentiable function on a closed interval $[a,b]$ ① find critical points, evaluate function there ② check endpoints.

Example ① find abs max/min of $2x^3 - 15x^2 + 24x + 7$ on $[0,3]$.

② $x^2 - 8$ on $[1,4]$.

③ $\sin(x)\cos(x)$ on $[0,\pi]$.