Thm $\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ $\quad$ $\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$

Proof (of $\sin^{-1}(x)$) $\quad y = \sin^{-1}(x)$

$$\sin(y) = x$$

$$\cos(y) \cdot y' = 1 \qquad y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}} \quad \square$$

Thm $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ $\quad$ $\frac{d}{dx}\left(\cot^{-1}(x)\right) = \frac{-1}{1+x^2}$

$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$ $\quad$ $\frac{d}{dx}\left(\operatorname{cosec}^{-1}(x)\right) = \frac{-1}{|x|\sqrt{x^2-1}}$

Proof (of $\tan^{-1}(x)$) $\quad y = \tan^{-1}(x)$

$$\tan(y) = x$$

$$\sec^2(y)\, y' = 1 \qquad y' = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2} \quad \square.$$

Example $\frac{d}{dx}\left(\tan^{-1}(e^{2x})\right) = \frac{1}{1+(e^{2x})^2} \cdot e^{2x} \cdot 2$

§3.9 Derivatives of exponentials and logs

recall $\quad f(x) = b^x \quad$ then $\quad f'(x) = m_b b^x$

$\qquad f(x) = e^x \quad$ then $\quad f'(x) = e^x$

Thm $f(x) = b^x$ then $f'(x) = \ln(b)\, b^x$.

Proof $f(x) = b^x = e^{\ln(b)x}$ $\qquad f'(x) = e^{\ln(b)x} \cdot \ln(b) = b^x \ln(b). \ \square.$

Example $f(x) = 3^{4x} = (3^4)^x$ $\qquad f'(x) = \ln(3^4)(3^4)^x = 4\ln(3)\, 3^{4x}.$

Thm $f(x) = \ln(x)$ then $f'(x) = \frac{1}{x}$

Proof $\quad y = \ln(x) \qquad e^y y' = 1$

$\qquad e^y = x \qquad y' = \frac{1}{e^y} = \frac{1}{x} \quad \square$

<u>Example</u> ① $f(x) = \log_b(x) = \dfrac{\ln(x)}{\ln(b)}$ $\quad f'(x) = \dfrac{1}{x\ln(b)}$

② $f(x) = x\ln(x)$, $f'(x) = \ln(x) + x\cdot\frac{1}{x} = \ln(x) + 1$

<u>Hyperbolic trig functions</u>

$\sinh(x) = \dfrac{e^x - e^{-x}}{2}$ $\quad \cosh(x) = \dfrac{e^x + e^{-x}}{2}$

$\dfrac{d}{dx}(\sinh(x)) = \cosh(x)$, $\dfrac{d}{dx}(\cosh(x)) = \sinh(x)$ <u>nb</u>: $\cosh^2 x - \sinh^2 x = 1$

<u>check</u>: $(\sinh(x))' = \left(\dfrac{e^x - e^{-x}}{2}\right)' = \frac{1}{2}(e^x - -e^{-x}) = \cosh(x)$

$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$ so $\dfrac{d}{dx}(\tanh(x)) = \dfrac{1}{\cosh^2 x}$, etc.

inverse functions: $\dfrac{d}{dx}(\sinh^{-1}(x)) = \dfrac{1}{\sqrt{x^2+1}}$ $\quad \dfrac{d}{dx}(\cosh^{-1}(x)) = \dfrac{1}{\sqrt{x^2-1}}$

$\dfrac{d}{dx}(\tanh^{-1}(x)) = \dfrac{1}{1-x^2}$

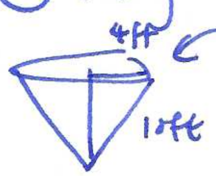
<u>§3.10 Related rates</u>

<u>Example</u> ① Balloon  $V = \frac{4}{3}\pi r^3$ inflate balloon: $V(t) = \frac{4}{3}\pi(r(t))^3$

$\dfrac{dV}{dt} = 4\pi r^2 \dfrac{dr}{dt}$ so if $\dfrac{dV}{dt} = 1\,ft^3/min$ and $r = 1ft$, then $\cancel{V=\frac{4}{3}}$

$1 = 4\pi 1^2 \cdot \dfrac{dr}{dt}$ so $\dfrac{dr}{dt} = \dfrac{1}{4\pi}$ ft/min.

② filling a conical tank



$10\,ft^3/min$ Q: how fast is the water rising when $h = 3ft$?

water in: $\dfrac{dV}{dt} = 10\,ft^3/min$, volume of cone: $V = \frac{1}{3}\pi h r^2$

need r in terms of h: $\dfrac{r}{h} = \dfrac{4}{10}$, i.e. $r = \frac{2}{5}h$

so $V = \frac{1}{3}\pi h\left(\frac{2}{5}h\right)^2 = \dfrac{4}{75}\pi h^3$, so $\dfrac{dV}{dt} = \dfrac{12}{75}\pi h^2 \dfrac{dh}{dt}$

so when $h = 5$

$10 = \frac{12}{75}\pi(5)^2\frac{dh}{dt}$

$\dfrac{dh}{dt} = \dfrac{10}{4\pi}$ ft/min