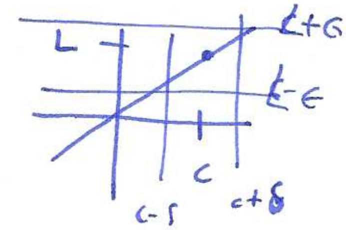
want to show: $|f(x)-5|$ close to zero when x close to 2

$|f(x)-5| = |2x+1-5| = |2x-4| = 2|x-2|$ $\quad$ x close to 2 $\Leftrightarrow$ $|x-2|$ close to 0.

Precise def$^{\underline{n}}$ let $f(x)$ be defined on an interval containing c, but not necessarily at c. We say $\lim_{x \to c} f(x) = L$ if for all $\epsilon > 0$ there is a $\delta > 0$ s.t. if $|c-x| < \delta$ then $|f(x) - L| < \epsilon$.



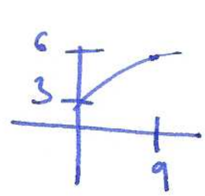
useful facts

Th$^{\underline{m}}$ for any constants k, c:
$$\lim_{x \to c} k = k$$
$$\lim_{x \to c} x = c$$

investigating limits: try
- drawing a picture
- calculating close values
- algebra

Example $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3}$ $\quad$ problem: plug in $x=9$ get $\frac{0}{0}$ undefined

- draw picture



looks like $f(9) = \cancel{\#}\ 6$

- calculate:

x	$\frac{x-9}{\sqrt{x}-3}$
8.9	5.983
9.1	6.016
8.99	5.998
9.01	6.002

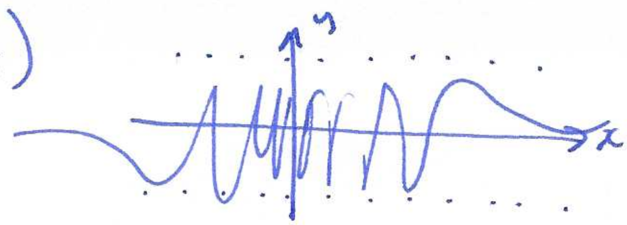
- algebra: difference of two squares

$$x - 9 = (\sqrt{x})^2 - (3)^2 = (\sqrt{x}-3)(\sqrt{x}+3)$$

$$\frac{x-9}{\sqrt{x}-3} = \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \sqrt{x}+3 \quad (x \neq 9)!!$$

so $\lim_{x \to 9} \frac{x-9}{\sqrt{x}-3} = \lim_{x \to 9} \sqrt{x}+3 = 6$

<u>Bad example</u> : no limit $f(x) = \sin\left(\frac{1}{x}\right)$

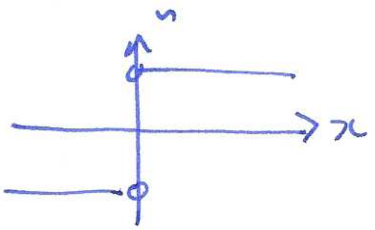


no limit at $x = 0$

note: $f\left(\frac{1}{2\pi n}\right) = \sin(2\pi n) = 0$

$f\left(\frac{1}{2\pi n + \frac{\pi}{2}}\right) = \sin\left(2\pi n + \frac{\pi}{2}\right) = 1$

<u>One sided limits</u> example $f(x) = \frac{x}{|x|}$ $f(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$



sometimes useful to distinguish left / right / two-sided limit.

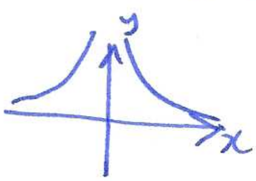
<u>notation</u> $\lim_{x \to 0+} f(x)$ means right limit (only consider $x > 0$)

$\lim_{x \to 0-} f(x)$ means left limit (only consider $x < 0$)

<u>note</u>: in order for the two sided limit to exist, left and right limits must exist and be equal.

<u>example</u> $f(x) = \frac{x}{|x|}$ $\left.\begin{array}{l} \lim_{x \to 0+} f(x) = +1 \\ \lim_{x \to 0-} f(x) = -1 \end{array}\right\} 1 \neq -1$ so $\lim_{x \to 0} f(x)$ DNE

<u>Example</u> $f(x) = \frac{1}{x^2}$



$\left.\begin{array}{l} \lim_{x \to 0+} \frac{1}{x^2} = +\infty \\ \lim_{x \to 0-} \frac{1}{x^2} = -\infty \end{array}\right\} \lim_{x \to 0} \frac{1}{x^2} = +\infty.$

<u>§2.3 Basic limit laws</u>

<u>Example</u>

$$\lim_{x \to 0} 2x + 2 = \lim_{x \to 0} 2x + \lim_{x \to 0} 2 = 2 \lim_{x \to 0} x + \lim_{x \to 0} 2 = 2$$

<u>Thm</u> assume that $\lim_{x\to c} f(x)$, $\lim_{x\to c} g(x)$ exist and are finite. Then:

1) sums : $\lim_{x\to c} (f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$

2) constant multiple : $\lim_{x\to c} kf(x) = k \lim_{x\to c} f(x)$ k constant (does not depend on x)

3) products : $\lim_{x\to c} (f(x)g(x)) = (\lim_{x\to c} f(x))(\lim_{x\to c} g(x))$

4) quotients : $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ as long as $\lim_{x\to c} g(x) \neq 0$

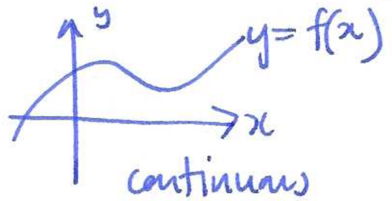
<u>warning</u> : these rules don't work if either $\lim_{x\to c} g(x)$ DNE
or $\lim_{x\to c} f(x)$ DNE.

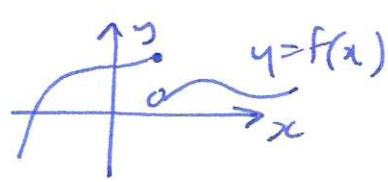
<u>Examples</u> $\lim_{x\to 3} x^2 = \lim_{x\to 3} x \cdot \lim_{x\to 3} x = 3.3 = 9.$

$\lim_{t\to 2} \frac{t+5}{2t} = \frac{\lim_{t\to 2} t+5}{\lim_{t\to 2} 2t} = \frac{7}{4}$

<u>§ 2.4 Limits and continuity</u>

<u>Example</u>





not continuous/ discontinuous

<u>Def^n</u> we say $f(x)$ is continuous at $x=c$ if $\lim_{x\to c} f(x) = f(c)$

If the limit DNE, is infinite, not equal to $f(c)$, then $f(x)$ is <u>not</u> continuous at $x=c$.