MTH 231 Calculus I

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- math tutoring 1S-214
- students w/ disabilities

Text: Calculus, early transcendentals, Rogawski + Adams.
HW: webworks.

§1.2 Linear and quadratic functions

recall: input set (domain) $\xrightarrow{\text{function}}$ output set (range)

examples: $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x^2$ ($\mathbb{R}$ = set of real numbers)

or $f(x) = x^2$.

example values: $0 \mapsto 0$, $-1 \mapsto 1$, $1 \mapsto 1$, $2 \mapsto 4$ etc.

notation: $f: \mathbb{R} \to \mathbb{R}$

name, domain/inputs, range/outputs.

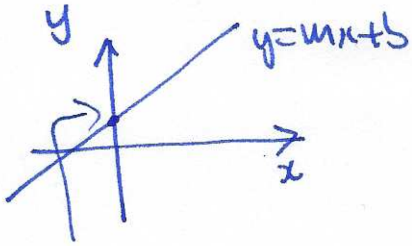
Examples · $+ : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, $(a,b) \mapsto a+b$

· evaluation at 0 $\{\text{functions } f: \mathbb{R} \to \mathbb{R}\} \to \mathbb{R}$, $f \mapsto f(0)$

key property: each input x gets mapped to one output value $f(x)$, not multiple output values...

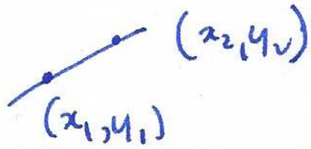
A linear function $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$ (m, b "constants", i.e. don't depend on x)

The graph of a linear function is a straight line. ②



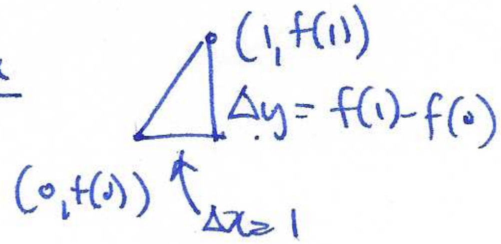
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$f(0) = m.0+b = b$. "y-intercept"



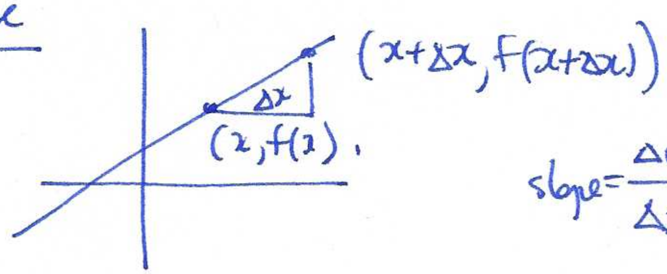
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

special case



$$\underset{m}{\text{slope}} = \frac{\Delta y}{\Delta x} = \cancel{f(x+\Delta x) - f(x)} \; \frac{f(1)-f(0)}{1-0} = \frac{m+b-b}{1} = m$$

general case

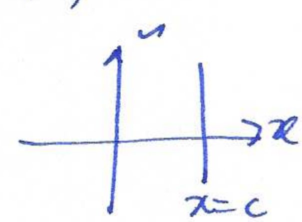


$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{(x+\Delta x) - x}$$

$$= \frac{m(x+\Delta x) + b - (mx+b)}{\Delta x} = \frac{m\Delta x}{\Delta x} = m$$

useful fact: a straight line has constant slope everywhere.

observations:
- $|m|$ large $\leftrightarrow$ steep slope
- $m = 0$ horizontal line
- $m > 0$ increasing (from left to right)
- $m < 0$ decreasing
- vertical lines not graphs of functions



equation	vs	function
relation between variables		a map from one set to another
$x = c$		e.g. $\cancel{f(x)}$ $f: \mathbb{R} \to \mathbb{R}$
$x^2 + y^2 = 1$		$x \mapsto f(x)$

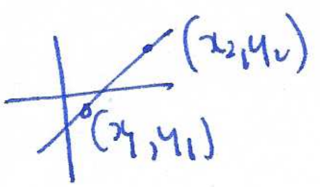
the <u>graph</u> of $f: \mathbb{R} \to \mathbb{R}$ is $y = f(x)$ (an equation!)

but not all equations come from functions

to deal with <u>any</u> straight line, use the <u>general/symmetric linear equation</u>

$ax+by=c$, at least one of a, b not zero.

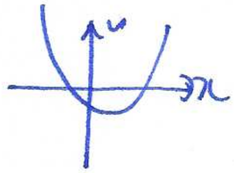
<u>useful technique</u>: find equation of line through two points



- find slope $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- line $y - y_1 = m(x - x_1)$ ← point slope formula for a line.

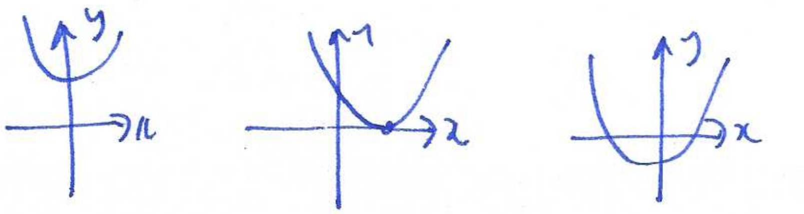
<u>quadratic functions</u> given by $f(x) = ax^2 + bx + c$ (a, b, c constants, do not depend on x)

graphs are parabolas



at most two distinct real solutions to $f(x) = 0$, given by

$$f(x) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$b^2 - 4ac < 0$ $\quad$ $b^2 - 4ac = 0$ $\quad$ $b^2 - 4ac > 0$

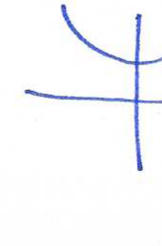
<u>useful techniques</u>

- factorization: if $ax^2 + bx + c = a(x - r_1)(x - r_2)$ then r_1, r_2 are solutions/roots.
- complete the square: any quadratic can be written as $a(x+b)^2 + c$

example: $x^2 + 2x + 3$

$(x+1)^2 + 2$

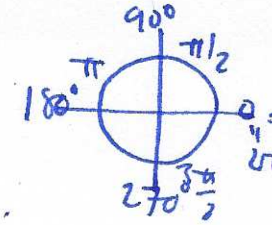
$x^2 + 2x + 1 + 2$



no solutions!

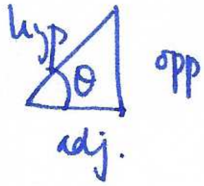
§1.4. Trig functions

- angles vs radians, radians win.



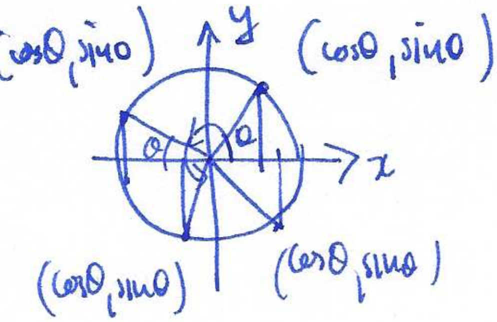
- angle in radians = distance travelled around the unit circle.

- right angled triangles



$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}$$

can extend these functions to be defined for all $\theta \in \mathbb{R}$



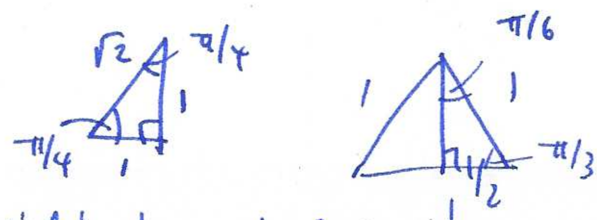
useful facts: $\sin(-\theta) = -\sin(\theta)$ (odd function)

$\cos(-\theta) = \cos\theta$ (even function)

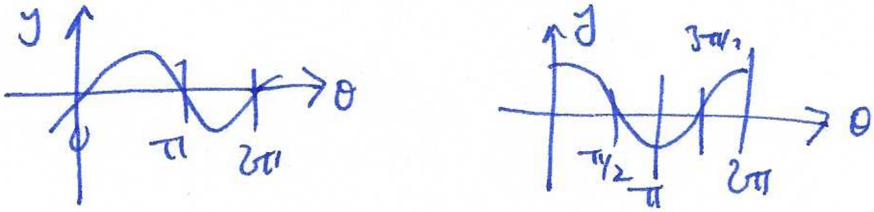
special values:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/\sqrt{2}$	0

this comes from special triangles



- graphs of $\sin(\theta)$, $\cos(\theta)$ are periodic of period 2π.



More trig functions

$$\tan(\theta) = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\sin\theta} = \operatorname{cosec}\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\frac{1}{\tan\theta} = \cot(\theta) = \frac{\cos\theta}{\sin\theta}$$

Trig identities Pythagorean identity:

$$\sin^2 x + \cos^2 x = 1$$