# Math 231 Calculus 1 Fall 24 Midterm 3b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a $3 \times 5$ index card of notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 3 | |
| Overall | |

(1) Find $\lim_{x \to 0} \dfrac{\sin(4x)}{1 - e^{-3x}}$.

$$\overset{l'H}{=} \lim_{x \to 0} \frac{4\cos(4x)}{3e^{-3x}} = \frac{4}{3}$$

(2) Find $\lim_{x\to 0} \dfrac{1-\cos(3x)}{\sqrt{1-2x^2}-1}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{3\sin(3x)}{\frac{1}{2}(1-2x^2)^{-1/2}\cdot(-4x)}$$

$$\overset{l'H}{=} \lim_{x\to 0} \frac{9\cos(9x)}{-\frac{1}{4}(1-2x^2)^{-3/2}\cdot(-4x)^2 + \frac{1}{2}(1-2x^2)^{-1/2}\cdot(-4)} = -\frac{9}{2}$$

(3) Consider the function $f(x) = \ln(x) - 2x$, for $x > 0$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = \frac{1}{x} - 2$ critical points $f'(x) = 0$

$\frac{1}{x} - 2 = 0 \Rightarrow x = \frac{1}{2}$

b) $f''(x) = -x^{-2}$

$f''\left(\frac{1}{2}\right) = -4 < 0 \Rightarrow x$ is local max.

(4) Consider the function $f(x) = \dfrac{x^3 - 8}{x}$. $= x^2 - 8x^{-1}$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptote at $x = 0$

$\lim_{x \to \pm\infty} f(x) = \pm\infty$ no horizontal asymptotes.

b) $f'(x) = 2x + 8x^{-2}$ inflection point $f''(x) = 0$

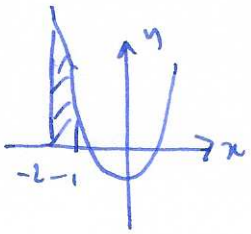
$f''(x) = 2 - 16x^{-3}$ $\quad 2 - \frac{16}{x^3} = 0 \Rightarrow x^3 = 8$

$x = 2$.

c) $f''(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$ f convex up

$f''(x) < 0$ on $(0, 2)$ f convex down.

(5) Find the area under the graph $y = 3x^2 - 2$ between $x = -2$ and $x = -1$.




$$\int_{-2}^{-1} 3x^2 - 2\,dx = \left[x^3 - 2x\right]_{-2}^{-1}$$

$$= (-1)^3 - 2(-1) - \left((-2)^3 - 2(-2)\right)$$

$$= -1 + 2 + 8 - 4 = 5$$

(6) Find the indefinite integral $\int 3\sqrt{x} - \frac{5}{x} + 2\cos(x)\ dx$.

$3 \cdot \frac{2}{3} x^{3/2} - 5\ln(x) + 2\sin(x) + C$

(7) Find the indefinite integral $\int x \sin(3 + 2x^2)\, dx$.

$u = 3 + 2x^2$

$\frac{du}{dx} = 4x$

$= \int x \sin(u) \frac{1}{4x}\, du$

$= \frac{1}{4} \int \sin(u)\, du$

$= -\frac{1}{4}\cos(u) + C = -\frac{1}{4}\cos\left(3 + 2x^2\right) + C$

(8) Find the definite integral $\int_0^1 e^x\sqrt{2+e^x}\,dx$.

$u = 2+e^x$ $\quad x=1 \Rightarrow u = 2+e$

$\frac{du}{dx} = e^x$ $\quad x=0 \Rightarrow u = 3.$

$$= \int_3^{2+e} e^x\, u^{1/2}\, \frac{dx}{du}\, du$$

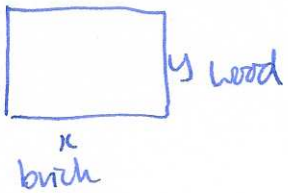
$$= \int_3^{2+e} e^x\, u^{1/2}\, \frac{1}{e^x}\, du$$

$$= \int_3^{2+e} u^{1/2}\, du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_3^{2+e}$$

$$= \frac{2}{3}\left((2+e)^{3/2} - (3)^{3/2}\right)$$

(9) You wish to enclose a rectangular garden with brick walls on two opposite sides, costing \$40/m and wooden fencing on the other two opposite sides sides, costing \$20/m. If the total area of the garden is $100\text{m}^2$, what are the dimensions which minimize the cost?




$$A = xy = 100 \Rightarrow y = \frac{100}{x}$$

$$C = 80x + 40y = 80x + \frac{4000}{x}$$

$$\frac{dC}{dx} = 80 - \frac{4000}{x^2}$$

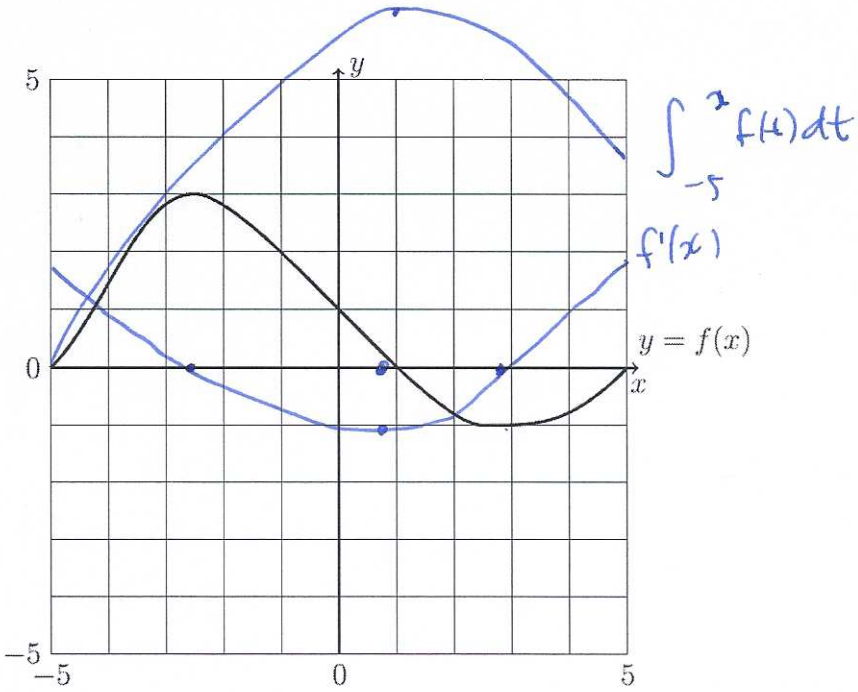
critical point $\frac{dC}{dx} = 0$

$$80 - \frac{4000}{x^2} = 0$$

$$x^2 = \frac{4000}{80} = 50 \Rightarrow x = \sqrt{50}$$

$$y = \frac{100}{\sqrt{50}}$$

(10) Consider the function $f(x)$ defined by the following graph.




(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. $x \approx 1$

(c) Sketch the graph of $\int_{-5}^{x} f(t)\ dt$.