Math 231 Calculus 1 Fall 24 Midterm 3a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x \to 0} \dfrac{e^{-2x} - 1}{\sin(3x)}$.

$$\overset{l'H}{=} \lim_{x \to 0} \frac{-2e^{-2x}}{3\cos(3x)} = \frac{-2}{3}$$

(2) Find $\lim_{x\to 0} \dfrac{1-\sqrt{1-3x^2}}{\cos(5x)-1}$.

$$\overset{L'H}{=} \lim_{x\to 0} \frac{-\frac{1}{2}\left(1-3x^2\right)^{-1/2}\cdot(-6x)}{-5\sin(5x)}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{4}\left(1-3x^2\right)^{-3/2}(-6x)^2 + -\frac{1}{2}\left(1-3x^2\right)^{-1/2}\cdot(-6)}{-25\cos(5x)} = -\frac{3}{25}$$

(3) Consider the function $f(x) = x - 3\ln(x)$ for $x > 0$.

(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a)

$f'(x) = 1 - \frac{3}{x}$ critical pnts $f'(x) = 0$

$1 - \frac{3}{x} = 0 \Rightarrow x = 3.$

b) $f''(x) = \frac{3}{x^2}$ $f''(3) = \frac{1}{3} > 0 \Rightarrow x = 3$ is local min

(4) Consider the function $f(x) = \dfrac{8 - x^3}{x}$. $= 8x^{-1} - x^2$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) vertical asymptotes : $x = 0$
horizontal asymptotes $\lim_{x \to \pm\infty} \frac{8-x^3}{x} = \pm\infty$ no horizontal asymptotes.

b) $f'(x) = -8x^{-2} - 2x^1$

$f''(x) = 16x^{-3} - 2$

inflection points $f''(x) = 0$

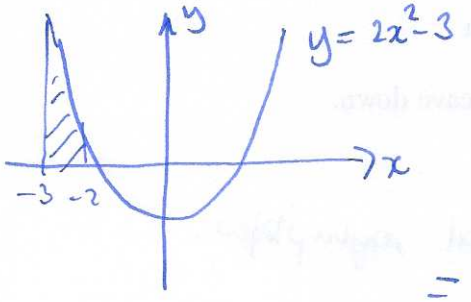
$\frac{16}{x^3} = 0$

$\frac{8}{1} = x^3$ $x = \pm\sqrt[3]{8}$ $x = 2$

$f''(x) > 0$ $(0, 2)$ f convex up

$f''(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$ f concave down.

(5) Find the area under the graph $y = 2x^2 - 3$ between $x = -3$ and $x = -2$.



$$\int_{-3}^{-2} 2x^2 - 3\,dx = \left[\frac{2}{3}x^3 - 3x\right]_{-3}^{(-2)}.$$

$$= \frac{2}{3}(-8) - 3(-2) - \left(\frac{2}{3}(-3)^3 - 3(-3)\right).$$

$$= -\frac{16}{3} + 6 + 18 - 9 = \frac{29}{3}$$

(6) Find the indefinite integral $\int 3\cos(x) - \frac{2}{x} + 5\sqrt{x}\, dx$.

$3\sin(x) - 2\ln(x) + 5 \cdot \frac{2}{3} x^{3/2} + C$

(7) Find the indefinite integral $\int x \sin(2+3x^2)\, dx$.

$u = 2+3x^2$

$\frac{du}{dx} = 6x$

$$= \int x \sin(u) \frac{dx}{du}\, du$$

$$= \int x \sin(u) \frac{1}{6x}\, du$$

$$= \frac{1}{6}\int \sin(u)\, du$$

$$= -\frac{1}{6}\cos(u) + C = -\frac{1}{6}\cos(2+3x^2) + C$$

(8) Find the definite integral $\int_0^1 e^x\sqrt{3+e^x}\,dx$.

$u = 3+e^x$ $\quad x=1 \Rightarrow u = 3+e$

$\frac{du}{dx} = e^x$ $\quad x=0 \Rightarrow u = 4$

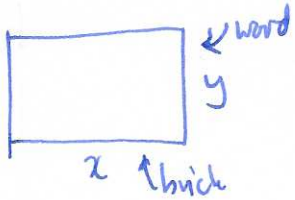
$$= \int_4^{3+e} e^x \cdot u^{1/2} \frac{dx}{du}\,du$$

$$= \int_4^{3+e} e^x \cdot u^{1/2} \cdot \frac{1}{e^x}\,du$$

$$= \int_4^{3+e} u^{1/2}\,du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_4^{3+e} = \frac{2}{3}\left((3+e)^{3/2} - (4)^{3/2}\right).$$

(9) You wish to enclose a rectangular garden with brick walls on two opposite sides, costing \$40/m and wooden fencing on the other two opposite sides sides, costing \$10/m. If the total area of the garden is 200m^2, what are the dimensions which minimize the cost?



$A = xy = 200 \Rightarrow y = \frac{200}{x}$

$C = 40 \times 2x + 10 \times 2y = 80x + 20y$

$C = 80x + 20 \cdot \frac{200}{x} = 80x + \frac{4000}{x}$

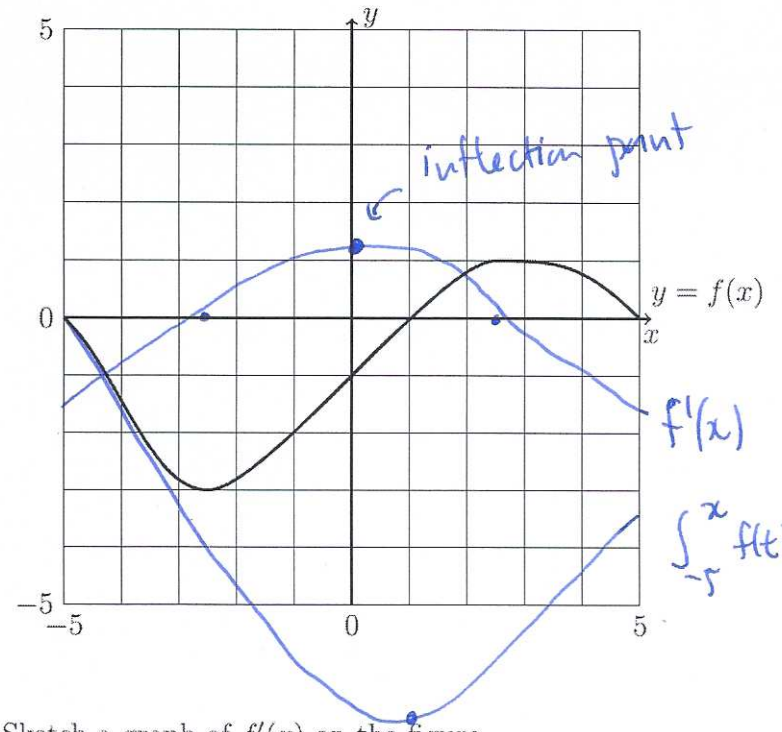
$\frac{dC}{dx} = 80 - \frac{4000}{x^2}$ critical point $\frac{dC}{dx} = 0$

$80 - \frac{4000}{x^2} = 0$

$x^2 = \frac{4000}{80} = 50 \quad x = \sqrt{50}$

$y = \frac{200}{\sqrt{50}}$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. $x=0$

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.