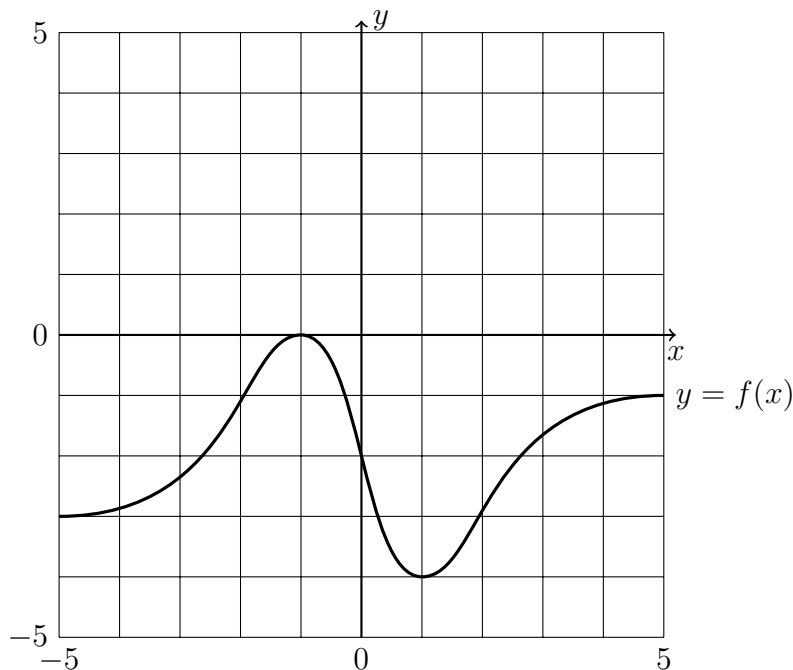


Math 231 Calculus 1 Fall 24 Sample Midterm 2

- (1) Consider the function $f(x)$ defined by the following graph.



- (a) Label all regions where $f'(x) < 0$.
 - (b) Label all regions where $f'(x) > 0$.
 - (c) Sketch a graph of $f'(x)$ on the figure.
 - (d) What is $\lim_{x \rightarrow \infty} f(x)$?
 - (e) What is $\lim_{x \rightarrow -\infty} f'(x)$?
 - (f) Label the approximate locations of all points of inflection.
- (2) Find the derivatives of the following functions
- (a) $x^4 e^{-2x^3}$
 - (b) $\frac{\sqrt{2x-1}}{4 - \cos(3x)}$
 - (c) $x^{\sqrt{x}}$
 - (d) $\sec(\sqrt{\ln(x)})$
 - (e) $\tan^{-1}(2/\sqrt[3]{x})$
 - (f) $\sin^{-1}(2-3x)$
- (3) Find the second derivatives of the functions in parts a), e) and f) above.

- (4) Use implicit differentiation to find the tangent line to the hyperbola $5x^2 - 3y^2 = -7$ at the point $(-1, -2)$.
- (5) Find $\frac{dy}{dx}$ for the implicit function $xy^2 - x^2y = \sin(x - y)$.
- (6) You inflate a spherical balloon at a rate of 6cm^3 per second. How fast is the area of the balloon increasing when the radius is 5cm ?
- (7) Use a linear approximation to estimate $\sqrt[4]{80}$. What is the percentage error?
- (8) Find all the critical points for the function $f(x) = e^{-x}(x^2 + 3x + 1)$. Use the first derivative test to identify them as local maxima or local minima.
- (9) Find the absolute maximum and minimum of the function $f(x) = 2x^2 - 5x - 3$ on the interval $[-2, 2]$.
- (10) Consider the function

$$f(x) = \frac{e^x}{1 - 4x^2}$$

- (a) Find all vertical and horizontal asymptotes of the function.
- (b) Find all critical points of the function.
- (c) Determine the intervals where $f(x)$ is increasing and decreasing.
- (d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
- (e) Sketch the function and label all relative maxima and minima.
- (11) A function $f(x)$ has derivative

$$f'(x) = \frac{1}{e^{x^2} + 1}.$$

Where on the interval $[1, 3]$ does it take its maximum value?

- (12) Compute the following limits. Show all work.

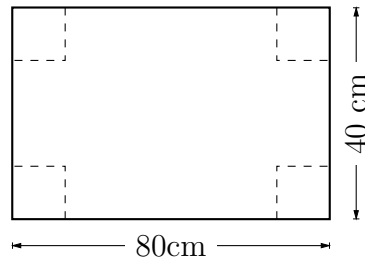
(a) $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 2}}{4 - 3x}$

(c) $\lim_{x \rightarrow 0} \sin(x) \ln(x)$

(b) $\lim_{x \rightarrow 0} \frac{\sin^{-1}(x/2)}{\cos^{-1}(3x)}$

(d) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin 3x} - \frac{1}{e^{3x} - 1} \right)$

- (13) You are swimming in the ocean 100m from the shore, and you wish to get to a point 200m along the beach from the closest point on the shore to you. If you can run four times as fast as you can swim, what is your fastest route? Assume you should swim in a straight line to some point along the shore.
- (14) We have a piece of cardboard that is 80cm by 40cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



- (15) Find the point on the line $y = 2x - 3$ which is closest to the point $(4, 3)$.
- (16) A piece of pipe is being carried down a hallway that is 10 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows down to 5 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?

