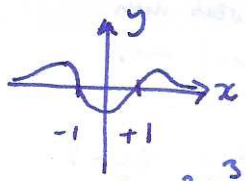
SMT2 Solutions



Q1 a) $(-1,1)$ b) $(-5,-1)\cup(1,5)$ c) d) -1 e) 0 f) $-2, 0, 2$

Q2 a) $4x^3e^{-2x^3} + x^4e^{-2x^3}\cdot(-6x^2) = 4x^3e^{-2x^3} - 6x^5e^{-2x^3}$

b) $$\frac{(4-\cos(3x))\cdot\frac{1}{2}(2x-1)^{-1/2}\cdot 2 - 3\sin(3x)\sqrt{2x-1}}{(4-\cos(3x))^2}$$

c) $$x^{\sqrt{x}} = e^{\sqrt{x}\ln(x)} = e^{\sqrt{x}\ln(x)}\cdot\left(\frac{1}{2}x^{-1/2}\ln(x) + \sqrt{x}\cdot\frac{1}{x}\right)$$

d) $$\sec\left(\sqrt{\ln x}\right)\tan\left(\sqrt{\ln x}\right)\cdot\frac{1}{2}(\ln x)^{-1/2}\cdot\frac{1}{x}$$

e) $$\frac{1}{1+(2x^{1/3})^2}\cdot\left(-\frac{2}{3}x^{-4/3}\right) = \frac{-2}{3x^{4/3}+12x^{2/3}} = -2\left(3x^{4/3}+12x^{2/3}\right)^{-1}$$

f) $$\frac{1}{\sqrt{1-(2-3x)^2}}\cdot(-3) = -3\left(1-(2-3x)^2\right)^{-1/2}$$

Q3 a) $12x^2e^{-2x^3} + 4x^3e^{-2x^3}\cdot(-6x^2) = 12x^2e^{-2x^3} - 24x^6e^{-2x^3}$

e) $$2\left(3x^{4/3}+12x^{2/3}\right)^{-2}\cdot\left(4x^{1/3}+8x^{-1/3}\right)$$

f) $$\frac{3}{2}\left(1-(2-3x)^2\right)^{-3/2}\cdot 2(2-3x)\cdot(-3)$$

Q4 $5x^2-3y^2=-7 \Rightarrow 10x-6yy'=0$ $y' = \frac{10x}{6y}$ at $(-1,-2)$ $y' = -\frac{5}{6}$

tangent line: $y+2 = -\frac{5}{6}(x+1)$

Q5 $xy^2-x^2y = \sin(x-y) \Rightarrow y^2 + x\cdot 2yy' - 2xy - x^2y' = \cos(x-y)\cdot(1-y')$

$y'(2xy-x^2+\cos(x-y)) = -y^2+2xy+\cos(x-y)$ $y' = \frac{-y^2+2xy+\cos(x-y)}{2xy-x^2+\cos(x-y)}$

Q6 $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ $\frac{dV}{dt} = 6$, $r=5$, $\frac{dr}{dt} = \frac{6}{4\cdot\pi\cdot 5^2}$

Q7 $f(x) = x^{1/4}$ $f'(x) = \frac{1}{4}x^{-3/4}$ $f(80) \approx f(81) - f'(81) = 3 - \frac{1}{4}\cdot\frac{1}{27}$

abs error $= \left|\sqrt[4]{80} - (3-\frac{1}{4\cdot 27})\right| \approx 4.818\times10^{-5}$ percentage error $= \frac{\text{abs error}}{(80)^{1/4}}\times 100 \approx 0.00144\%$

Q8 $f'(x) = -e^{-x}(x^2+2x+1) + e^{-x}(2x+3) = e^{-x}(-x^2+2)$ critical points $\pm\sqrt{2}$

f' : $-$ | $+$ | $-$; $-\sqrt{2}$, $+\sqrt{2}$.

$-\sqrt{2}$ local min

$+\sqrt{2}$ local max

Q9 $f(x) = 2x^2 - 5x - 3$ $\quad f'(x) = 4x - 5$ critical point $\frac{5}{4}$ check $f(-2) = 15$ → abs max.

$f(\frac{5}{4}) = -\frac{49}{8}$ → abs min. $\quad f(2) = -5$

Q10 $f(x) = \dfrac{e^x}{1-4x^2}$ $\quad f'(x) = e^x(1-4x^2)^{-1} + e^x \cdot -(1-4x^2)^{-2} \cdot (-8x)$

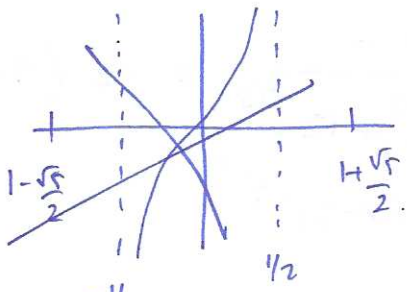
$e^x(1-4x^2)^{-1}$

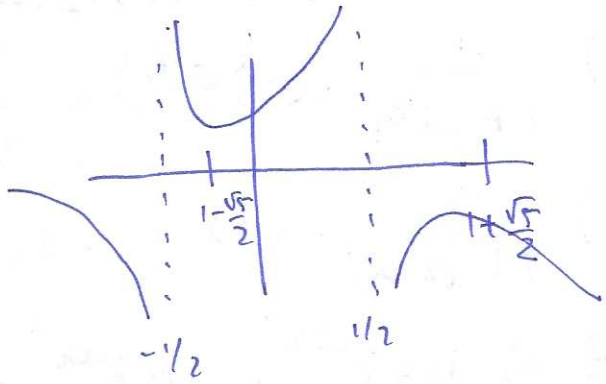
$= e^x \dfrac{1-4x^2+8x}{(1-4x^2)^2}$ critical points: $\dfrac{-8 \pm \sqrt{64+16}}{-8} = 1 \pm \frac{\sqrt{5}}{2}$

a) $x = \pm\frac{1}{2}$

b) $1 \pm \frac{\sqrt{5}}{2}$

c) f' : $-$ | $+$ | $-$; $1-\frac{\sqrt{5}}{2}$, $1+\frac{\sqrt{5}}{2}$.





d) slip – too long ux 1st derivative t/f.

Q11 $f'(x) > 0$, f increasing so max value at $x = 3$.

Q12 a) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{3x^2-2}}{4-3x} \Big/ x = \lim\limits_{x \to -\infty} \dfrac{\sqrt{3-2/x^2}}{4/x + 3} = \dfrac{\sqrt{3}}{3}$.

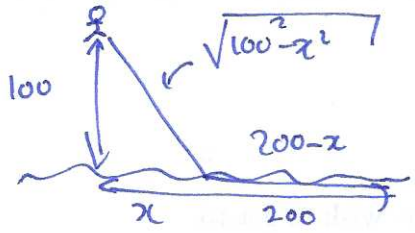
b) $\lim\limits_{x \to 0} \dfrac{\sin^{-1}(x/2)}{\cos^{-1}(3x)} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{\frac{1}{\sqrt{1-(x/2)^2}}}{\frac{-1}{\sqrt{1-(3x)^2}}} = \lim\limits_{x \to 0} -\dfrac{\sqrt{1-9x^2}}{\sqrt{1-x^2/4}} = -1$

c) $\lim\limits_{x \to 0} \dfrac{\ln(x)}{\operatorname{cosec}(x)} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{1/x}{-\operatorname{cosec}(x)\cot(x)} = \underbrace{\lim\limits_{x \to 0} -\dfrac{\sin(x)}{x}}_{=-1} \cdot \underbrace{\lim\limits_{x \to 0} \tan(x)}_{=0} = 0$

d) $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1 - \sin 3x}{(\sin 3x)(e^{3x}-1)} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{3e^{3x} - 3\cos 3x}{3\cos 3x\,(e^{3x}-1) + \sin 3x \cdot 3e^{3x}} \overset{l'H}{=} \lim\limits_{x \to 0} \dfrac{3e^{3x} + 3\sin 3x}{-\sin 3x \cdot 3(e^{3x}-1) + \cos 3x \cdot 3e^{3x} + \cos 3x\,(3e^{3x}) + \sin 3x\, 3e^{3x}}$

$= \dfrac{3}{6} = \dfrac{1}{2}$

Q13



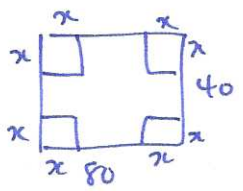
time $T = \frac{\sqrt{100^2+x^2}}{V} + \frac{200-x}{4V}$

$\frac{dT}{dx} = \frac{1}{2V}\left(100^2+x^2\right)^{-1/2}\cdot(+2x) - \frac{1}{4V}$

critical point $\frac{dT}{dx}=0$: $\frac{4x}{\sqrt{100^2+x^2}} = 1$ $\quad 4x = \sqrt{100^2+x^2}$ $\quad 16x^2 = 100^2+x^2$ $\quad 17x^2 = 10^2$

$x = \frac{10}{\sqrt{17}}$.

Q14



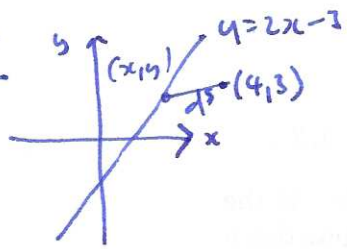
$V = x(80-x)(40-x) = x(x^2-120x+3200)$

$= x^3 - 120x^2 + 3200x$

$\frac{dV}{dx} = 3x^2 - 240x + 3200$

$x = \frac{240 \pm \sqrt{240^2 - 12\times 3200}}{6.} \approx 16.91$

Q15



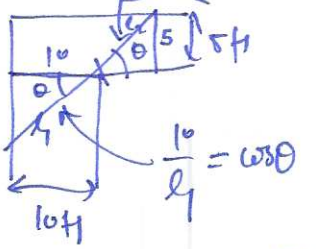
$d^2 = (x-4)^2 + (y-3)^2$

$= (x-4)^2 + (2x)^2 =$

$\frac{d}{dx}(d^2) = 2(x-4) + 8x$

critical point : $10x = 8$ $\quad x = 4/5$ $\quad \left(\frac{4}{5}, -\frac{7}{5}\right)$.

Q16



length: $L = l_1 + l_2 = \frac{10}{\cos\theta} + \frac{5}{\sin\theta} = 10(\cos\theta)^{-1} + 5(\sin\theta)^{-1}$

$\frac{dL}{d\theta} = -10(\cos\theta)^{-2}\cdot(-\sin\theta) + -5(\sin\theta)^{-2}\cdot\cos\theta$

critical point : $\frac{10\sin\theta}{\cos^2\theta} = \frac{5\cos\theta}{\sin^2\theta}$ $\quad 2\sin^3\theta = \cos^3\theta$ $\quad \frac{\sin^3\theta}{\cos^3\theta} = \frac{1}{2}$ $\quad \tan\theta = \sqrt[3]{1/2}$

$\theta \approx 0.6709$

$L \approx 20.81$