Math 231 Calculus 1 Fall 24 Midterm 1b

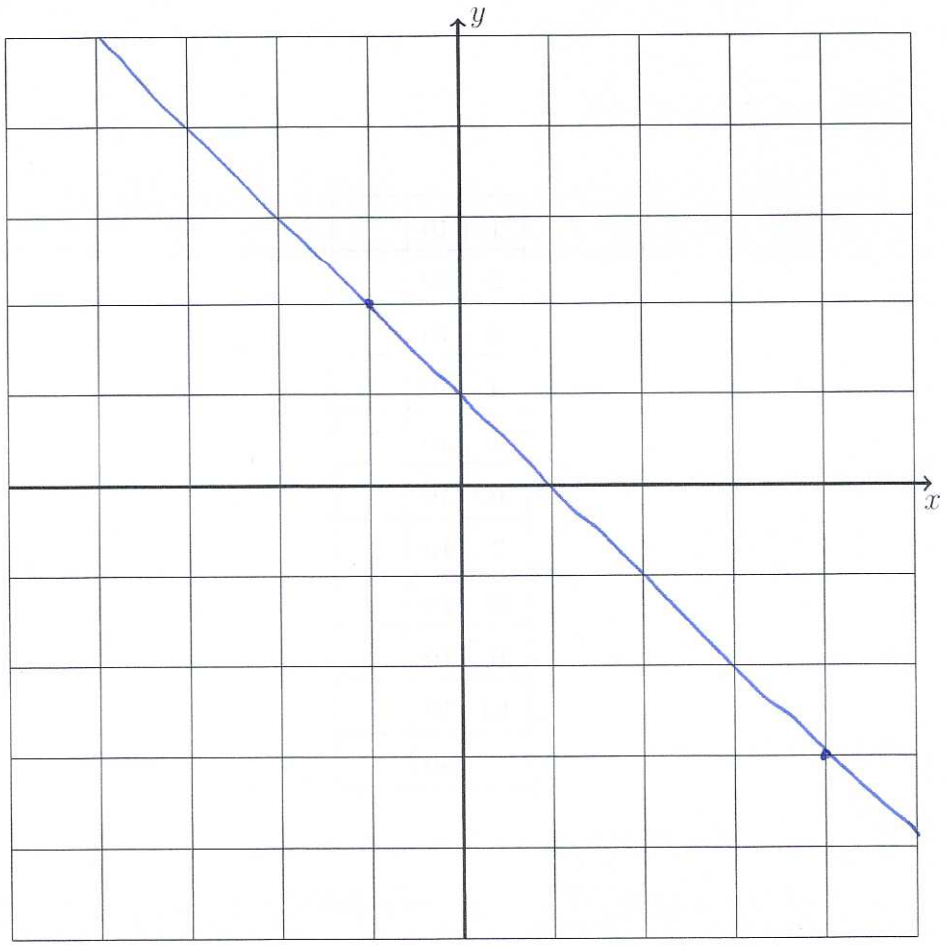
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(4, -3)$ and $(-1, 2)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



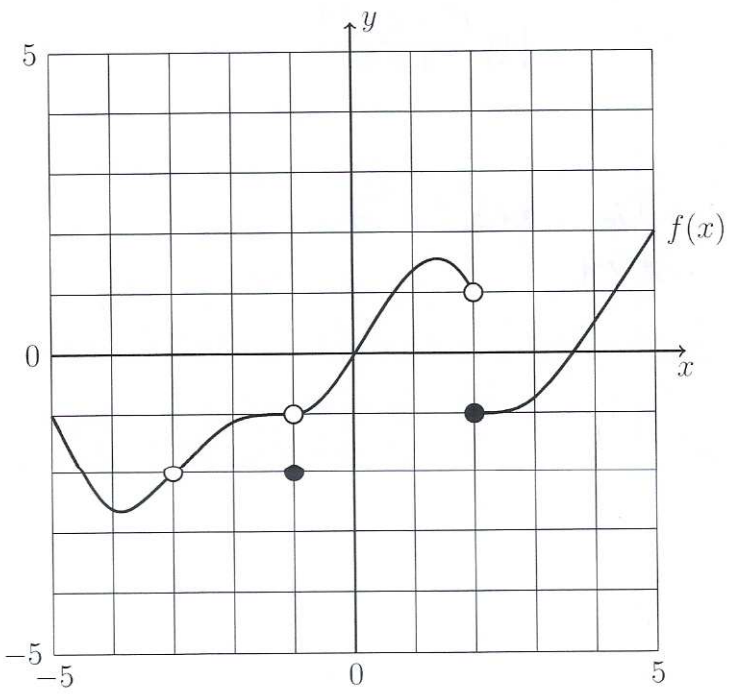
slope $m = \dfrac{-1-4}{2-(-3)} = \dfrac{-5}{5} = -1$

$y - y_0 = m(x - x_0)$

$y - 2 = -(x+1)$

$y = -x + 1$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to -1-} f(x)$ -1

(b) $\lim_{x\to -1} f(x)$ -1

(c) $\lim_{x\to 2+} f(x)$ -1

(d) $\lim_{x\to 2} f(x)$ DNE

(e) $\lim_{x\to -3} f(x)$ -2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$$

$= \lim_{x \to 4} \frac{(x-4)(x+3)}{(x-4)} = \lim_{x \to 4} x+3 = 7$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$$

$$= \lim_{x\to 4} \frac{2-\sqrt{x}}{(2-\sqrt{x})(2+\sqrt{x})} = \lim_{x\to 4} \frac{1}{2+\sqrt{x}} = \frac{1}{4}.$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = 2x - x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h) - (x+h)^2 - (2x - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{2x + 2h - x^2 - 2xh - h^2 - 2x + x^2}{h}$$

$$= \lim_{h \to 0} 2 - 2x - h = 2 - 2x$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{3-2x}{\sqrt{2+4x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{3}{x}-2}{\sqrt{\frac{2}{x^2}+4}} = \frac{-2}{\sqrt{4}} = -1$$

(7) Find the first and second derivatives of $f(x) = \cos(x) + x^3 - 2/\sqrt{x}$.

$f(x) = \cos(x) + x^3 - 2x^{-1/2}$

$f'(x) = -\sin(x) + 3x^2 + x^{-3/2}$

$f''(x) = -\cos(x) + 6x - \frac{3}{2}x^{-5/2}$

(8) Find the first and second derivatives of $f(x) = \sqrt[3]{x} - \frac{x}{e^x}$.

$$f'(x) = \frac{1}{3}x^{-2/3} - \frac{e^x(x)' - (e^x)'\cdot x}{(e^x)^2} = \frac{1}{3}x^{-2/3} - \frac{e^x - xe^x}{(e^x)^2}$$

$$= \frac{1}{3}x^{-2/3} - \frac{1-x}{e^x}$$

$$f''(x) = -\frac{2}{9}x^{-5/3} - \frac{(e^x)(1-x)' - (1-x)(e^x)'}{(e^x)^2}$$

$$= -\frac{2}{9}x^{-5/3} - \frac{-e^x - (1-x)e^x}{(e^x)^2}$$

$$= -\frac{2}{9}x^{-5/3} + \frac{2+x}{e^x}$$

(9) Find the first and second derivatives of $f(x) = \sin^2(x)$.

$f(x) = \sin x \cdot \sin x$

$f'(x) = (\sin x)' \sin x + \sin x (\sin x)'$

$= \cos x \cdot \sin x + \sin x \cos x = 2 \sin x \cos x$

$f''(x) = 2(\sin x)' \cos x + 2(\sin x)(\cos x)'$

$= 2\cos^2 x - 2\sin^2 x$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.