Math 231 Calculus 1 Fall 24 Midterm 1a

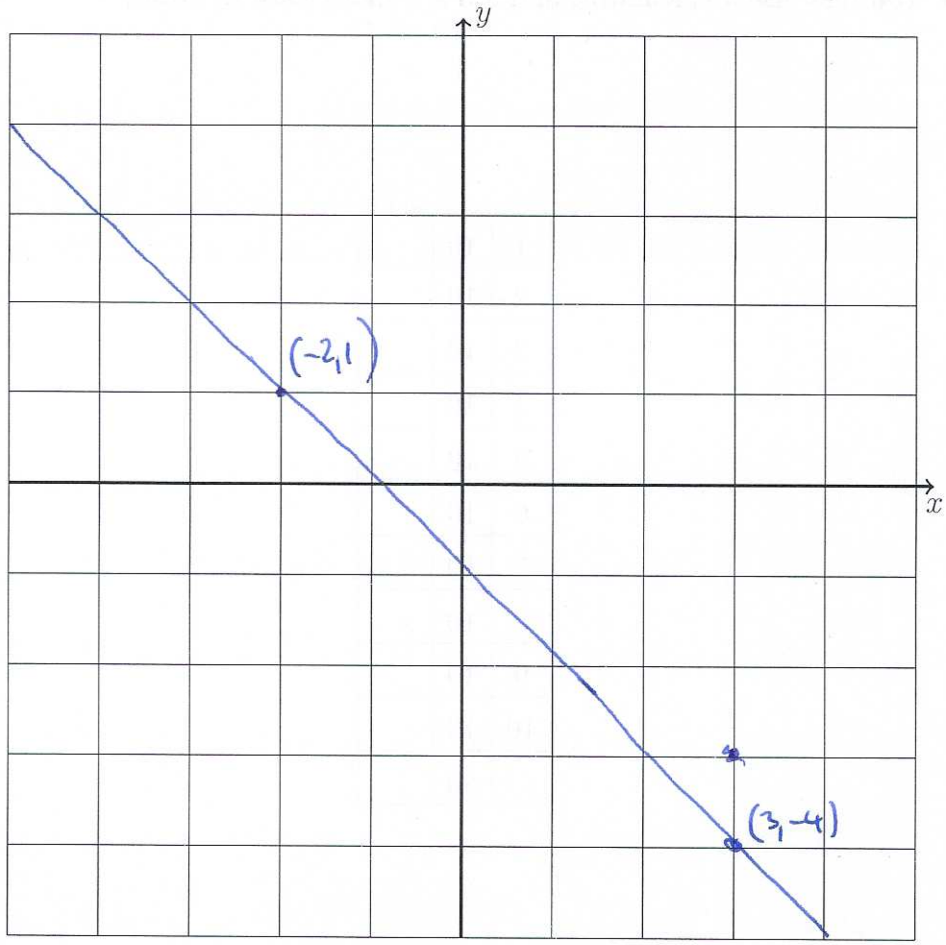
Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(3, -4)$ and $(-2, 1)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.



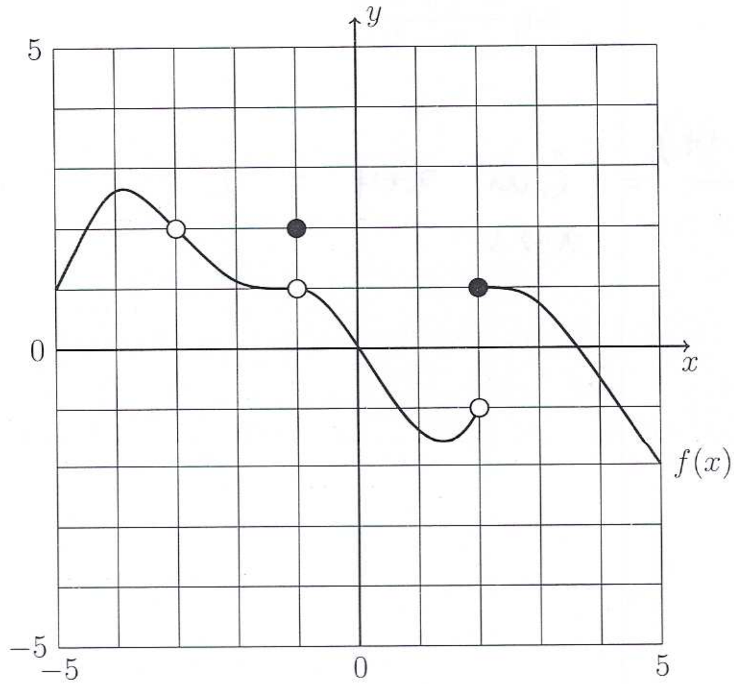
slope $m = \dfrac{-4-1}{3-(-2)} = \dfrac{-5}{5} = -1$

equation $y - y_0 = m(x - x)$

$y - 1 = -1(x+2)$

$y = -x - 1$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to -1^-} f(x)$ 1

(b) $\lim_{x\to -1} f(x)$ 1

(c) $\lim_{x\to 2+} f(x)$ 1

(d) $\lim_{x\to 2} f(x)$ DNE

(e) $\lim_{x\to -3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 3} \frac{x^2 + x - 12}{x - 3}$$

$$= \lim_{x \to 3} \frac{(x-3)(x+4)}{(x-3)} = \lim_{x \to 3} x+4 = 7$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 9} \frac{9 - x}{\sqrt{x} - 3}$$

$$= \lim_{x \to 9} \frac{(3-\sqrt{x})(3+\sqrt{x})}{\sqrt{x} - 3} = \lim_{x \to 9} -(3+\sqrt{x}) = -6$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 3x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h - \cancel{x^2} + \cancel{3x}}{h}$$

$$= \lim_{h \to 0} 2x + h - 3 \quad = \quad 2x - 3$$

(6) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{2x-4}{\sqrt{3x^2+4}}$$

$$= \lim_{x\to\infty} \frac{2-4/x}{\sqrt{3+4/x^2}} = \frac{2}{\sqrt{3}}$$

(7) Find the first and second derivatives of $f(x) = \sin(x) - x^4 + 3/\sqrt{x}$. $3x^{-1/2}$

$f'(x) = \cos(x) - 4x^3 - \frac{3}{2}x^{-3/2}$

$f''(x) = -\sin(x) - 12x^2 + \frac{9}{4}x^{-5/2}$

(8) Find the first and second derivatives of $f(x) = \dfrac{x}{e^x} - \sqrt[4]{x}$.

$$f'(x) = \frac{e^x \cdot 1 - xe^x}{(e^x)^2} - \frac{1}{4}x^{-3/4} = \frac{1}{e^x} - \frac{x}{e^x} - \frac{1}{4}x^{-3/4}$$

$$f''(x) = \frac{e^x \cdot (0) - e^x \cdot 1}{(e^x)^2} - \frac{e^x \cdot 1 - e^x \cdot x}{(e^x)^2} + \frac{3}{16}x^{-7/4}$$

$$= \frac{-1}{e^{x}} - \frac{1}{e^{x}} + \frac{x}{e^x} + \frac{3}{16}x^{-7/4}$$

$$= (x-2)e^{-x} + \frac{3}{16}x^{-7/4}$$

(9) Find the first and second derivatives of $f(x) = \cos^2(x)$.

$f(x) = \cos(x)\cos(x)$

$f'(x) = (\cos(x))'\cos x + \cos x(\cos x)'$

$= -\sin x\cos x + \cos x \cdot (-\sin x) = -2\sin x\cos x$

$f''(x) = -2(\sin x)'\cos x - 2\sin x(\cos x)'$

$= -2\cos^2 x + 2\sin^2 x$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.