

Math 231 Calculus 1 Fall 24 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

(a) $3x^4 - \frac{2}{\sqrt[3]{x^4}} + \sec(x)$

(b) $f(x) = \frac{x^2 - x}{\ln(2x - 1)}$

(c) $f(x) = e^{-4x} \cos(3 - 2x)$

(d) $f(x) = \sqrt[4]{e^{-\cos(3x)} + 2}$

(2) Evaluate the following integrals.

(a) $\int \frac{4}{x^3} - 2 \sin(x) - e^x \, dx$

(b) $\int \frac{(3x - 2)^2}{\sqrt{x^3}} \, dx$

(c) $\int_0^{\pi/6} \cos^3(2x) \sin(2x) \, dx$

(d) $\int \frac{1}{9 + x^2} \, dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or “does not exist” (DNE). Justify your answers.

(a) Find $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 - x - 6}$.

(b) Find $\lim_{x \rightarrow 0} \frac{1 - e^{4x}}{\sin(3x)}$.

(c) Find $\lim_{x \rightarrow 0^+} x^{\sin(2x)}$.

(d) Find $\lim_{x \rightarrow 0} \frac{1}{\sin^2(x)} - \frac{1}{x^2}$.

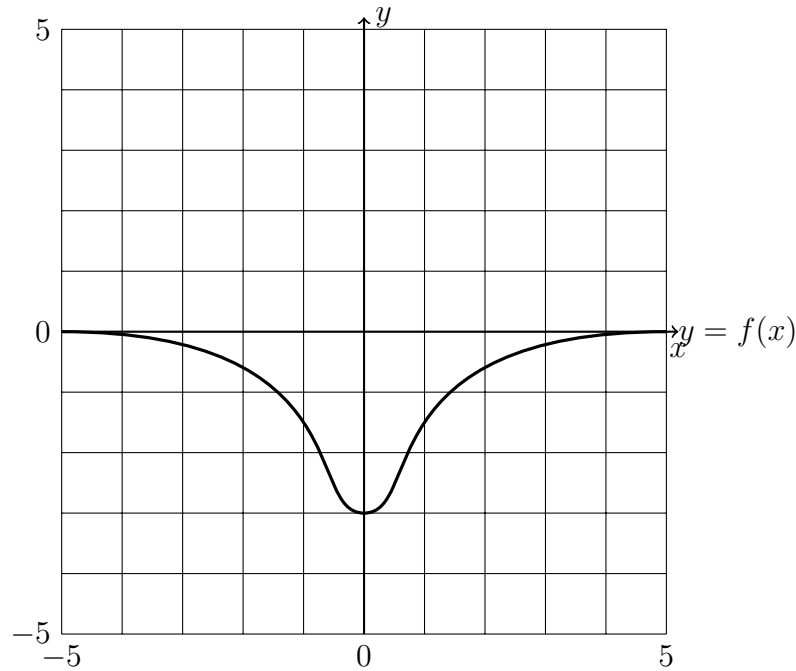
(4) Consider $f(x) = 27x - x^3$.

(a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.

(b) Give the interval(s) for which f is increasing.

- (c) Give the intervals for which f is concave up, and for which it is concave down.
- (d) Decide which critical points are maxima, minima, or neither.
- (e) Sketch the graph of $f(x)$.

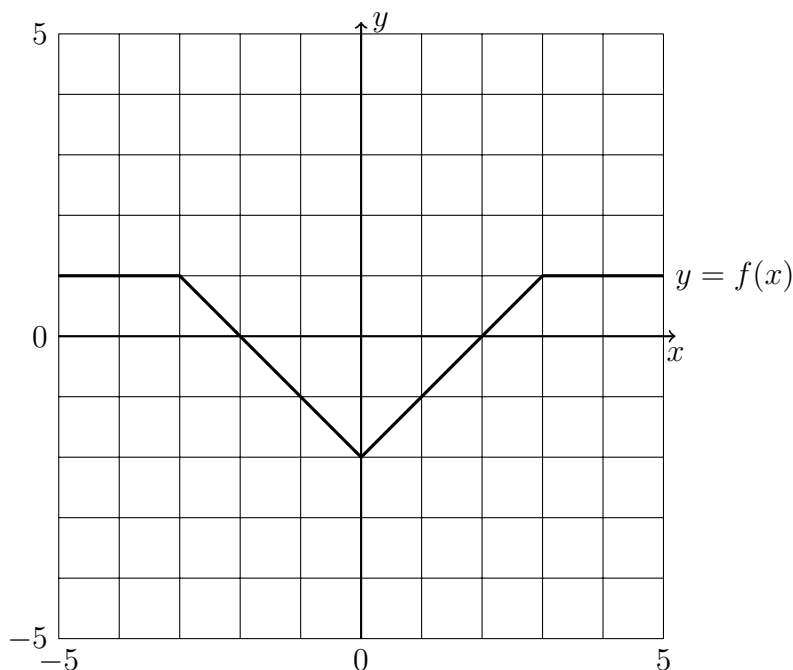
(5) Consider the function $f(x)$ defined by the following graph.



- (a) Label all regions where $f(x) < 0$.
 - (b) Label all regions where $f'(x) > 0$.
 - (c) Sketch a graph of $f'(x)$ on the figure.
- (6) Consider $f(x) = \frac{2}{2-x}$.
- (a) Sketch the graph of $f(x)$ showing any asymptotes.
 - (b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.
 - (c) Sketch the tangent line at $x = -1$ on your graph.
- (7) Let $f(x) = x - \frac{1}{x}$. Find the derivative *using the limit definition of the derivative*. Do not use L'Hôpital's rule. Show all your work.

- (8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y + 3xy^2 - 2x = -12$ at the point $(-1, -2)$.

- (9) Sketch the graph of $\int_{-5}^x f(t)dt$, where $f(x)$ is shown below.



- (10) A region in the plane is bounded by the x -axis, the graph $y = 4 - x^2$, and the lines $x = -1$ and $x = 1$.
- Sketch the region (shading it in) and label the boundaries.
 - Find the area of the region.
- (11) You blow up a spherical balloon at the rate of $4\text{ in}^3/\text{s}$. How fast is the volume growing when $r = 2\text{ in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$.)
- (12) Use linear approximation to estimate $\sqrt[3]{7}$. Use your calculator to find the exact value, and find the absolute and percentage errors.
- (13) What's the closest point on the line $y = 4 - 2x$ to the point $(4, 2)$?