SF solutions

(1)

Q1 a) $12x^3 - 2\cdot -\frac{4}{3}x^{-7/3} + \sec(x)\tan(x)$

b) $\dfrac{\ln(2x-1)(2x-1) - \frac{1}{2x-1}\cdot(x^2-x)}{\ln(2x-1)^2}$.

c) $-4e^{-4x}\cos(3-2x) + e^{-4x}\cdot -\sin(3-2x)\cdot(-2)$

d) $\frac{1}{4}\left(e^{-\cos(3x)}+2\right)^{-3/4}\cdot\left(e^{-\cos(3x)}\cdot\sin(3x)\cdot 3\right)$

Q2 a) $\dfrac{4\cdot x^{-2}}{-2} + 2\cos(x) - e^{x} + C$

b) $\int \cancel{3x}^{-1/2}\dfrac{9x^2-6x+4}{x^{3/2}}dx = \int 9x^{1/2} - 6x^{-1/2} + 4x^{-3/2}dx = 9\cdot x^{3/2}\cdot\frac{2}{3} - 6x^{1/2}\cdot 2 + 4x^{-1/2}\cdot -2 + C$

c) $\int_0^{\pi/6}\cos^3(2x)\sin(2x)dx$ $\quad u = \cos(2x)$, $\frac{du}{dx} = -\sin(2x)\cdot 2$ $\quad \int_1^{1/2} u^3\cancel{\sin(2x)}\cdot\frac{1}{-\sin(2x)\cdot 2}dx = \int_1^{1/2} -\frac{1}{2}u^3 du$

$= \left[-\frac{1}{8}u^4\right]_1^{1/2} = -\frac{1}{8}\cdot\frac{1}{16} + \frac{1}{8}$.

d) $\int\frac{1}{9+x^2}dx = \frac{1}{9}\int\frac{1}{1+(x/3)^2}dx$ $\quad u = \frac{x}{3}$, $\frac{du}{dx} = \frac{1}{3}$ $\quad \frac{1}{9}\int\frac{1}{1+u^2}\cdot 3du = \frac{1}{3}\tan^{-1}(u) + c = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

Q3 a) $\overset{l'H}{=}\lim_{x\to -2}\frac{1}{2x-1} = -\frac{1}{5}$

b) $\overset{l'H}{=}\lim_{x\to 0}\frac{-4e^{4x}}{3\cos(3x)} = \frac{-4}{3}$

c) $\overset{\circledast}{=}\lim_{x\to 0+} e^{\ln(x)\sin(2x)} = e^{\lim_{x\to 0+}\ln(x)\sin(2x)}$. $\to = \lim_{x\to 0+}\frac{\ln(x)}{\operatorname{cosec}(2x)} \overset{l'H}{=} \lim_{x\to 0+}\frac{1/x}{-\operatorname{cosec}(2x)\cot(2x)\cdot 2}$

$= \lim_{x\to 0+}\frac{\sin 2x\tan 2x}{2x} \overset{l'H}{=} \lim_{x\to 0+}\frac{2\cos 2x\tan 2x + \sin 2x\sec^2(2x)\cdot 2}{2} = \frac{0}{2} = 0$ so $\circledast = e^0 = 1$.

d) $\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\sin^2 x} \overset{l'H}{=} \lim_{x\to 0}\frac{2x - 2\sin x\cos x}{2x\sin^2 x + x^2 2\sin x\cos x}$ $\overset{l'H}{=} \lim_{x\to 0}\frac{2 - 2\cos^2 x + 2\sin^2 x}{2\sin^2 x + 2x2\sin x\cos x + 4x\sin x\cos x + x^2 2\cos^2 x - x^2 2\sin^2 x}$.

$\overset{l'H}{=} \lim_{x\to 0}\frac{4\cos x\cdot\sin x + 4\sin x\cos x}{4\sin x\cos x + 6\sin x\cos x + 6x\cos^2 x - 6x\sin^2 x + 4x\cos^2 x + x^2\cdot 4\cos x\sin x - 4x\sin^2 x - x^2\cdot 4\sin x\cos x}$

$\overset{l'H}{=} \lim_{x\to 0}\frac{8\cos^2 x - 8\sin^2 x}{14\cos^2 x - 18\sin^2 x}$

$= \lim_{x\to 0}\frac{2x - \sin 2x}{2x\sin^2 x + x^2\sin 2x} \overset{l'H}{=} \lim_{x\to 0}\frac{2 - \cancel{2\sin x\cos x}\ 2\cos 2x}{2\sin^2 x + 2x\cdot 2\sin x\cos x + 2x\sin 2x + x^2\cdot 2\cos 2x}$.

$= \lim_{x\to 0}\frac{2 - 2\cos 2x}{2\sin^2 x + 4x\sin 2x + 2x^2\cos 2x}$ $\overset{l'H}{=} \lim_{x\to 0}\frac{4\sin 2x}{4\sin x\cos x + 4\sin 2x + 8x\cos 2x + 4x\cos 2x + 2x^2\cdot -\sin 2x\cdot 2}$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{8\cos 2x}{4\cos 2x + 8\cos 2x + 8\cos 2x + 8x\cdot -2\sin 2x + 4\cos 2x - 8x\sin 2x - 4\cos 4x} = \frac{8}{20}$$

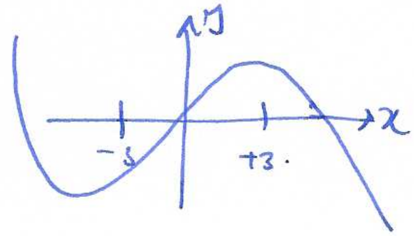
Q4 $f(x) = 27x - x^3$ $\quad f'(x) = 27 - 3x^2 = 3(9 - x^2)$ critical points $x = \pm 3$.

b) increasing for $(-3, 3)$

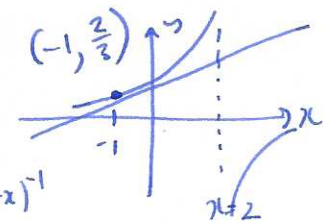
c) $f''(x) = -6x$ $\quad$ concave up $(-\infty, 0)$
down $(0, \infty)$

d) -3 local min $\quad +3$ local max

e)



Q6 a) $(-1, \frac{2}{3})$



$f(x) = \frac{2}{2-x}$
$= 2(2-x)^{-1}$

$f'(x) = -2(2-x)^{-2}\cdot(-1)$

$f'(-1) = \frac{2}{9}$

b) $y - \frac{2}{3} = \frac{2}{9}(x+1)$

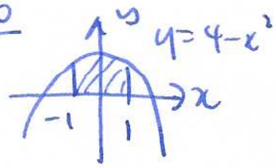
Q7 $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{x+h - \frac{1}{x+h} - (x - \frac{1}{x})}{h} = \lim_{h\to 0} 1 + \frac{\frac{1}{x} - \frac{1}{x+h}}{h}$

$= \lim_{h\to 0} 1 + \frac{1}{h}\frac{x+h-x}{x(x+h)} = \lim_{h\to 0} 1 + \frac{1}{x(x+h)} = 1 + \frac{1}{x^2}$.

Q8 $x^2y + 3xy^2 - 2x = -12$ $\quad 2xy + x^2y' + 3y^2 + 3x2yy' - 2 = 0$

$x = -1,\ y = -2$: $\quad 4 + y' + 12 + 12y' - 2 = 0$ $\quad y' = \frac{14}{13}$ $\quad y + 2 = \frac{14}{12}(x+1)$.

Q10



$$\int_{-1}^{1} 4 - x^2\,dx = \left[4x - \frac{1}{3}x^3\right]_{-1}^{1} = 4 - \frac{1}{2} - \left(-4 + \frac{1}{3}\right) = 8 - \frac{2}{3}.$$

Q11 $V = \frac{4}{3}\pi r^3$ $\quad \underbrace{\frac{dV}{dt}}_{4} = 4\pi \underbrace{r^2}_{2^2} \frac{dr}{dt}$ $\quad \frac{dr}{dt} = \frac{1}{4\pi}$ in/s.

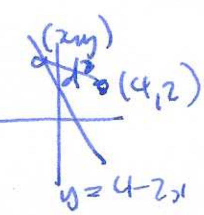
Q12 $f(x) = x^{1/3}$
$f'(x) = \frac{1}{3}x^{-2/3}$

$f(x + \Delta x) \simeq f(x) + f'(x)\Delta x$

$\sqrt[3]{7} \approx \sqrt[3]{8} + \frac{1}{3}\frac{1}{2^2}\cdot -1 = 2 - \frac{1}{12}$

abs error $= \left|\sqrt[3]{7} - \frac{23}{12}\right|$ $\quad$ percentage error $= \frac{\left|\sqrt[3]{7} - \frac{23}{12}\right| \times 100}{\sqrt[3]{7}}$.

Q13

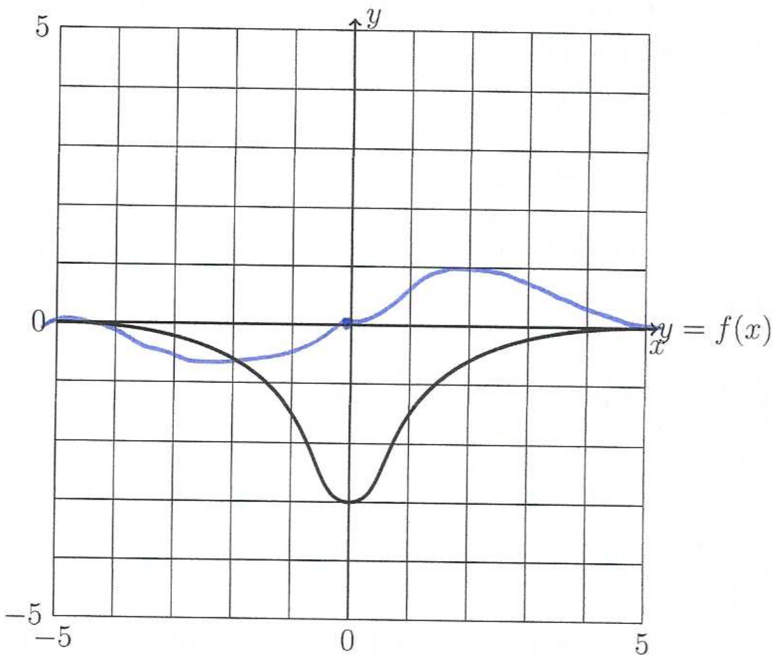


$d^2 = (x-4)^2 + (y-2)^2 = (x-4)^2 + (\underbrace{4-2x-2}_{-2x+2})^2 = \begin{array}{l} x^2 - 8x + 16 \\ + 4x^2 - 8x + 4 \end{array} = 5x^2 - 16x + 20$

$\frac{d}{dx}(d^2) = 10x - 16$ $\quad x = \frac{16}{10} = \frac{8}{5}$

(c) Give the intervals for which f is concave up, and for which it is concave down.
(d) Decide which critical points are maxima, minima, or neither.
(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.



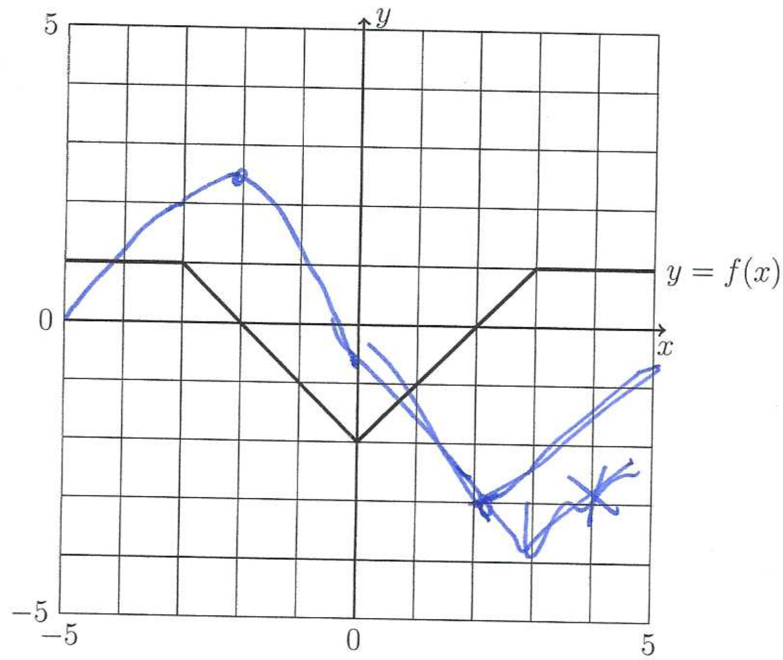
(a) Label all regions where $f(x) < 0$. $(-5, 5)$
(b) Label all regions where $f'(x) > 0$. $(0, 5)$
(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{2}{2-x}$.
(a) Sketch the graph of $f(x)$ showing any asymptotes.
(b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.
(c) Sketch the tangent line at $x = -1$ on your graph.

(7) Let $f(x) = x - \frac{1}{x}$. Find the derivative *using the limit definition of the derivate.* Do not use L'Hôpital's rule. Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y + 3xy^2 - 2x = -12$ at the point $(-1, -2)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.



(10) A region in the plane is bounded by the x-axis, the graph $y = 4 - x^2$, and the lines $x = -1$ and $x = 1$.

(a) Sketch the region (shading it in) and label the boundaries.

(b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $4\text{in}^3/\text{s}$. How fast is the volume growing when $r = 2\text{in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$.)

(12) Use linear approximation to estimate $\sqrt[3]{7}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) What's the closest point on the line $y = 4 - 2x$ to the point $(4, 2)$?