Last time

n-simplex

$\Delta = [v_0, v_1, \ldots, v_n]$

vertices ordered!

Δ-complex

· Δ, collection of simplices

F, face pairings

X topological space

geometric realization

$X = \bigsqcup \Delta_i \big/ \sim$

recall quotient topology

(Y, T) topological space

set

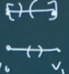
open sets

$\sim$ equivalence relation on Y

$q: Y \to \bar{Y} = Y/\sim$ $\quad (\bar{Y}, \bar{T})$ — $U \subseteq \bar{Y}$ s.t. $q^{-1}(U)$ open

Examples

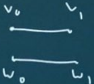
① $\Delta = \{[v_0,v_1]\}$



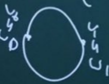
$F = \{[v_0] \sim [v_1]\}$

$X =$  $\cong S^1$

② $\Delta = \{[v_0,v_1],[w_0,w_1]\}$



$F = \{[v_0] \sim [w_0], [v_1] \sim [w_1]\}$

$X =$  $\cong S^1$

③ $\Delta = \{[v_0,v_1],[w_0,w_1]\}$

$F = \{[v_0,v_1] \sim [w_0,w_1]\}$

$X =$ 

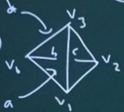
④ $X =$ any graph is a Δ-complex

$\Delta = \{[v_0,v_1],[w_0,w_1],[u_0,u_1]\}$

$F = \{[v_0] \sim [u_1], [w_0] \sim [u_0], [v_1] \sim [u_1], [w_1] \sim [v_1]\}$

Examples

$X = S^2$ 

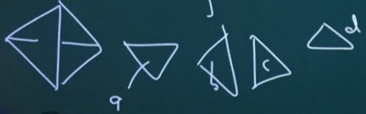
① 

$\Delta = \{ [a_0, a_1, a_2], [b_0, b_1, b_2], [c_0, c_1, c_2], [d_0, d_1, d_2] \}$

$F = \{ [a_0, a_1] \sim [b_0, b_1],$

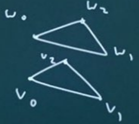
$\vdots$

$\}$



② $\Delta = \{ [v_0, v_1, v_2], [w_0, w_1, w_2] \}$

$F = \{ [v_0, v_1] \sim [w_0, w_1],$
$[v_0, v_2] \sim [w_0, w_2],$
$[v_1, v_2] \sim [w_1, w_2] \}$



X  $\cong S^2$

$S^n \subseteq \mathbb{R}^{n+1}$

↑ unit vectors

$\Delta =$ 2 n-simplices

$$\{[v_0, v_1, \dots, v_n], [w_0, w_1, \dots, w_n]\}$$

$$F = \{ [v_0, v_1, \dots, v_{n-1}] \sim [w_0, w_1, \dots, w_{n-1}], [v_0, v_1, \dots, v_{n-2}, v_n] \sim [w_0, w_1, \dots, w_{n-2}, w_n], \}$$

notation $[v_0, \dots, \hat{v}_i, \dots, v_n]$

means write down list of vertices but omit v_i

$$F = \{ [v_0, \dots, \hat{v}_i, \dots, v_n] \sim [w_0, \dots, \hat{w}_i, \dots, w_n] \}$$

v $\qquad$ w

surface $S \Leftarrow$ locally homeomorphic to $\mathbb{R}^2$

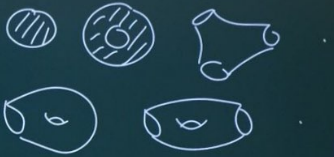
<u>Thm</u> Classification of surfaces

<u>closed</u> $\Leftrightarrow$ compact, no boundary

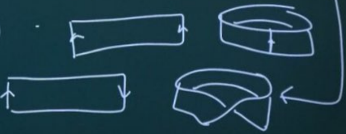
orientable

S^2 $\quad T^2 = S^1 \times S^1$ $\quad S_2$ $\quad S_3$

with boundary – cut holes in closed surfaces



Möbius band



Non-orientable surfaces

- compact no boundary

$\mathbb{RP}^2$, Klein bottle, ..

construction take orientable surface connect sum $\mathbb{RP}^2$

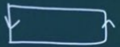
$S^2 \subseteq \mathbb{R}^3$

antipodal map $x \longmapsto -x$

can take $S^2/\sim$ $\quad x \sim -x$

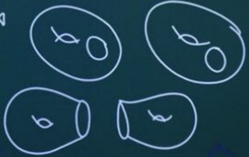
$=$  $B^2 \subseteq \mathbb{R}^2$

antipodal map on $\partial B^2 = S^1$

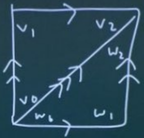


Connect sum $S_1 \# S_2$

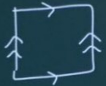
Ex $\quad T^2 \# T^2$.

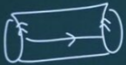


$T^2 = S^1 \times S^1$



S^1





$$\mathbb{Z}_{10} \xrightarrow{\alpha} \mathbb{Z}_2 \oplus \mathbb{Z}_5$$

$\{0,1,2,3,\ ,9\},+$ $\quad \{(0,0),(0,1),\cdots,(0,4),$
$(1,0),(1,1),\ ,(1,4)\}$

$$1 \xmapsto{\alpha} (1,1)$$

$2 = 1+1$ $\quad \alpha(2) = \alpha(1)+\alpha(1) = (1,1)+(1,1) = (2,2)$
$= (0,2)$

$3 = 2+1$ $\quad \alpha(3) = \alpha(2)+\alpha(1) = (0,2)+(1,1) = (1,3)$

$4 = 3+1$

$\vdots$

$1 \mapsto (1,0)$
homomorphism
not isomorphism

$1 \mapsto (1,3)$ also iso

Q2 size $36 = 2 \times 2 \times 3 \times 3$

recall $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \neq \mathbb{Z}_4$

$\mathbb{Z}_p \oplus \mathbb{Z}_q = \mathbb{Z}_{pq}$ p,q coprime

$\mathbb{Z}/36\mathbb{Z} = \mathbb{Z}_{36} \cong \mathbb{Z}_4 \oplus \mathbb{Z}_9$

$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_9 \cong \mathbb{Z}_2 \oplus \mathbb{Z}_{18}$

$\mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3 \cong \mathbb{Z}_{12} \oplus \mathbb{Z}_3$

$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$

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$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$

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