

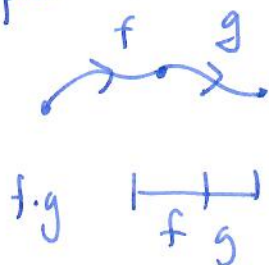
• Sample points from $S^2 \subseteq \mathbb{R}^3$ in R, uniformly

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```
X <- {
  a <- rnorm(100)
  b <- rnorm(100)
  c <- rnorm(100)
  x = a / sqrt(a^2 + b^2 + c^2)
  y = b / sqrt(a^2 + b^2 + c^2)
  z = c / sqrt(a^2 + b^2 + c^2)
  cbind(x, y, z)
}
```

• Example $\mathbb{Z}^3 \hookrightarrow \mathbb{Z}^3 \hookrightarrow \mathbb{Z}^3 \hookrightarrow \mathbb{Z}^2 \hookrightarrow \mathbb{Z}^1$

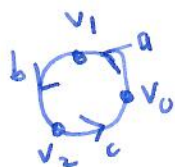
• path concatenation



$$f \cdot g(x) = \begin{cases} f(2x) & 0 \leq x \leq \frac{1}{2} \\ g(2x-1) & \frac{1}{2} \leq x \leq 1 \end{cases}$$

ck a $f(1) = g(0)$.

• $f: S^1 \rightarrow S^1$
 $\theta \mapsto 3\theta$



$H_0: \mathbb{Z} \rightarrow \mathbb{Z}$
 $[v_0] \mapsto [v]$

$$f[v_0] = [f(v_0)] = [v] \text{ so map is } \mathbb{Z} \xrightarrow{\cong} \mathbb{Z}$$

$H_1: \mathbb{Z} \rightarrow \mathbb{Z}$
 $[a+b+c] \mapsto [e]$

$$f[a+b+c] = [f(a)+f(b)+f(c)] = [e+e+e] = 3[e]$$

so map is $\mathbb{Z} \xrightarrow{\times 3} \mathbb{Z}$