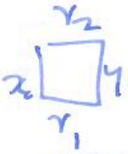
Last time $\pi_1(X, x_0) \leftarrow$ group of loops based at x_0 in X.

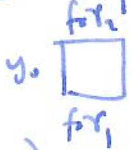
Prop^n: $f: X \to Y$, $x_0 \mapsto y_0$ induces $f_*: \pi_1(X, x_0) \to \pi_1(Y, y_0)$

$$[\gamma] \longmapsto [f \circ \gamma]$$

Proof check. well defined $\gamma_1 \sim \gamma_2 \Rightarrow \exists\ F: I \times I \to X$



then $f \circ F: I \times I \to Y$



✓

• homomorphism $f_*([\gamma_1][\gamma_2]) = [f \circ \gamma_1][f \circ \gamma_2] = [(f \circ \gamma_1) \cdot (f \circ \gamma_2)] = [f \circ (\gamma_1 \cdot \gamma_2)]$

✓ D

Q: how to find $\pi_1(X, x_0)$?

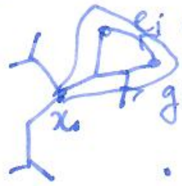
X Δ-complex $\rightsquigarrow$ presentation of $\pi_1(X, x_0) = \langle g_1 g_2, \cdots g_k \mid r_1, \dots r_\ell \rangle$

$g_1 g_2, \cdots g_k$: generators; $r_1, \dots r_\ell$: relations.

Example $\mathbb{Z}^2 = \langle a, b \mid ab = ba \rangle$

$aba^{-1}b^{-1} = 1$

Method: Let T be a maximal tree in $X^{(1)}$ containing x_0 ← also contains every other vertex!



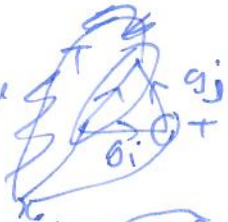
• every edge $e_i \in X^{(1)} \setminus T$ corresponds to a generator g_i

case: all edges of a simplex contained in T # T tree.

• two edges:

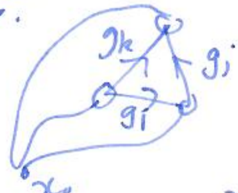
$\Rightarrow g_i = 1$.

• one edge



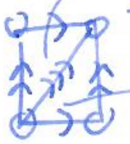
$\Rightarrow g_i = g_j$

• no edges



$g_i g_j = g_k$

Example $T^2 = S^1 \times S^1$



$g_2 g_1 = g_3$

$g_1 g_2 = g_3$

$\langle g_1, g_2, g_3 \mid g_1 g_2 = g_3,\ g_2 g_1 = g_3 \rangle$

$= \langle g_1, g_2 \mid g_1 g_2 = g_2 g_1 \rangle$

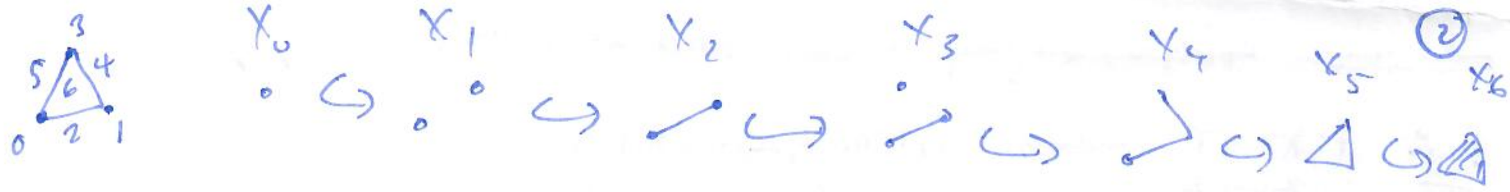
$\Rightarrow = \mathbb{Z}^2$.

van Kampen $(X, x_0) \vee (Y, y_0) = Z \ni z_0$ then $\pi_1(Z, z_0) = \pi_1(X) * \pi_1(Y)$

Simple case. Free product: $\pi_1(X) = \langle a_i \mid b_i \rangle$ $\pi_1(Y) = \langle c_i \mid d_i \rangle$

$\pi_1(X) * \pi_1(Y) = \langle a_i, c_i \mid b_i, d_i \rangle$

Example $S^1 \vee S^1$ $\langle a, b \mid \rangle$ $T^2 \vee T^2 = \langle a, b, c, d \mid ab = ba,\ cd = dc \rangle$



②

	X_0	X_1	X_2	X_3	X_4	X_5	X_6
$H_0(X_0) \cong \mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
$H_0(X_1) \cong \mathbb{Z}^2$		$\mathbb{Z}^2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
			$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
				$\mathbb{Z}^2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
						$\mathbb{Z}$	$\mathbb{Z}$
							$\mathbb{Z}$

$H_1(X)$							
	0	0	0	0	0	$\mathbb{Z}$	0
		0	0	0	0	0	0
			0	0	0	0	0
				0	0	0	0
					0	0	0
						1	0
							0