recall $X \rightsquigarrow$ chain complex $\rightsquigarrow H_k(X)$

want $f: X \to Y$ $\qquad f_*: H_k(X) \to H_k(Y)$

example $i\ X \subset Y$ inclusion of Δ-complexes induces map $i_*: H_k(X) \to H_k(Y)$

Defn A map $f_{\#}: C_k(X) \to C_k(Y)$ is a chain map if $\partial f = f \partial$, i.e.

$$\begin{tikzcd} \cdots \arrow[r] & C_{k+1}(X) \arrow[r, "\partial"] \arrow[d, "f"] & C_k(X) \arrow[r] \arrow[d, "f"] & \cdots \\ \cdots \arrow[r] & C_{k+1}(Y) \arrow[r, "\partial"] & C_k(Y) \arrow[r] & \cdots \end{tikzcd}$$

commutes.

Prop a chain map $f: C_k(X) \to C_k(Y)$ induces a map on homology $f_*: H_k(X) \to H_k(Y)$

Proof $H_k(X) \ni [z]$ $\quad z$ cycle in C_k i.e. $\partial z = 0$

$$\begin{array}{ccc} H_k(X) & \ni & [z] \\ \downarrow & & \downarrow \\ H_k(Y) & & [f(z)] \end{array}$$

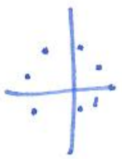
as $f(z)$ also a cycle.

$$\begin{tikzcd} y & z \arrow[r, mapsto, "\partial"] & 0 \\ C_{k+1}(X) \arrow[r] \arrow[d, "f"] & C_k(X) \arrow[r] \arrow[d, "f"] & C_{k-1}(X) \arrow[d, "f"] \\ C_{k+1}(Y) \arrow[r, "\partial"] & C_k(Y) \arrow[r, "\partial"] & C_{k-1}(Y) \\ & f(z) \arrow[r, mapsto] & \partial f(z) = 0 \end{tikzcd}$$

check well defined: spse $[z + \partial y]$ for $y \in C_{k+1}(X)$

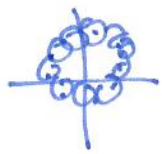
$\mapsto [f(z + \partial y)] = [f(z) + f(\partial y)] = [f(z)] + [f(\partial y)] = [\partial f(y)]$ ✓ $\square$.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad = 0$

Persistent homology data set $X = \{x_i\}_{i=1}^n \subseteq \mathbb{R}^d$



idea: define $X_r = \bigcup_{i=1}^n B(x_i, r)$

and consider homology $H_k(X_r)$

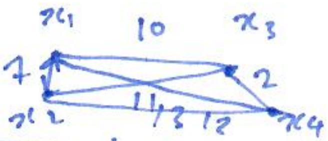


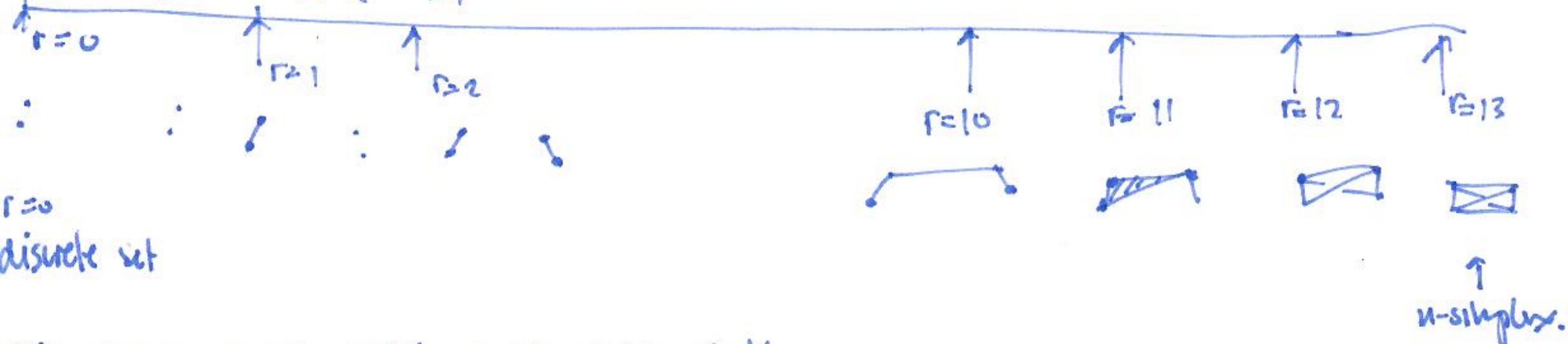
problem hard to compute $\bigcup_{i=1}^n B(x_i, r)$

solution use Rips complex X_r = • connect x_i, x_j if $d(x_i, x_j) \leq r$

• fill in all triangle, tetrahedra etc.

Example





$X_0 \subset X_1 \subset X_2 \subset X_{10} \subset X_{11} \subset X_{12} \subset X_{13}$.